

Graph representation

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Outline

We discuss two ways graphs can be represented using data structures.

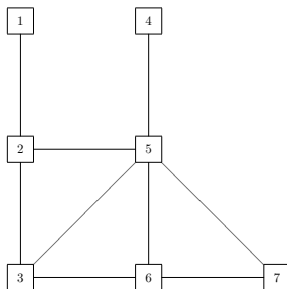
Representation of graphs

Two ways to represent a graph — adjacency lists or an adjacency matrix.

Adjacency list The graph G is represented by an array of $|V(G)|$ linked lists, with each list containing the neighbours of a vertex.

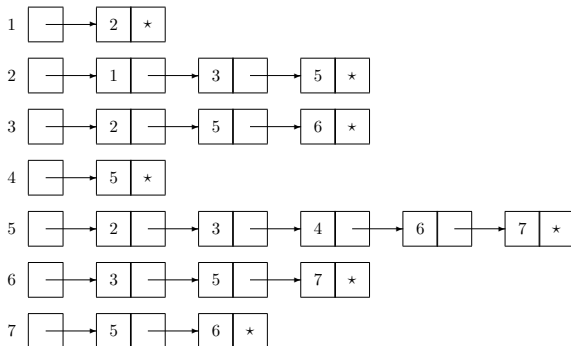
Adjacency list example

Suppose we have the following graph, G_4 :



Adjacency list example

Using an adjacency list representation, we would represent G_4 as follows:



This requires two list elements for each edge and thus the space required is $O(|V(G)| + |E(G)|)$.

Adjacency matrix

Adjacency matrix The *adjacency matrix* of a graph G is a $V \times V$ matrix A where the rows and columns are indexed by the vertices and such that $A_{ij} = 1$ if and only if vertex i is adjacent to vertex j .

Adjacency matrix

Using an adjacency matrix, for graph G_4 we have the following

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

The adjacency matrix representation uses $O(V^2)$ space.

Choice of representation

- ▶ We have seen that for an adjacency list, the space required is $O(|V(G)| + |E(G)|)$
- ▶ And for an adjacency matrix, the space required is $O(V^2)$.

- ▶ For a *sparse* graph E is much less than V^2 , and hence we would normally prefer the adjacency list representation
- ▶ However, for a *dense* graph E is close to V^2 and the adjacency matrix representation is preferred.

More on the two representations

- ▶ For small graphs or those without weighted edges it is often better to use the adjacency matrix representation anyway.
- ▶ It is also easy and more intuitive to define adjacency matrix representations for directed and weighted graphs.
- ▶ However your final choice of representation depends precisely what questions you will be asking.

Graph questions

Consider how you would answer the following questions in both representations (in particular, how much time it would take):

- ▶ Is vertex v adjacent to vertex w in an undirected graph?
- ▶ What is the out-degree of a vertex v in a directed graph?
- ▶ What is the in-degree of a vertex v in a directed graph?