Fuzzy Logic

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Outline

- Fundamental fuzzy concepts
- Fuzzy propositional and predicate logic
- Fuzzification
- Defuzzification
- Fuzzy control systems
- Types of fuzzy algorithms
- Applications of fuzzy logic

Introduction

- Fuzzy concepts first introduced by Zadeh in the 1960s and 70s
- Traditional computational logic and set theory is all about
  - true or false
  - zero or one
  - in or out (in terms of set membership)
  - black or white (no grey)
- Not the case with fuzzy logic and fuzzy sets!

Basic Concepts

- Approximation ("granulation")
  - A colour can be described precisely using RGB values, or it can be approximately described as "red", "blue", etc.
- Degree ("graduation")
  - Two different colours may both be described as "red", but one is considered to be more red than the other
- Fuzzy logic attempts to reflect the human way of thinking

Terminology

- Fuzzy set
  - A set X in which each element y has a grade of membership $\mu_X(y)$ in the range 0 to 1, i.e. set membership may be partial
    e.g. if *cold* is a fuzzy set, exact temperature values might be mapped to the fuzzy set as follows:
    - 15 degrees → 0.2 (slightly cold)
    - 10 degrees → 0.5 (quite cold)
    - 0 degrees → 1 (totally cold)

- Fuzzy relation
  - Relationships can also be expressed on a scale of 0 to 1
    e.g. degree of *resemblance* between two people

Terminology (cont'd)

- Fuzzy variable
  - Variable with (labels of) fuzzy sets as its values
- Linguistic variable
  - Fuzzy variable with values that are words or sentences in a language

    e.g. variable *colour* with values *red*, *blue*, *yellow*, *green*…
- Linguistic hedge
  - Term used as a modifier for basic terms in linguistic values

    e.g. words such as *very*, *a bit*, *rather*, *somewhat*, etc.

Formal Fuzzy Logic

- Fuzzy logic can be seen as an extension of ordinary logic, where the main difference is that we use fuzzy sets for the membership of a variable
- We can have fuzzy propositional logic and fuzzy predicate logic
- Fuzzy logic can have many advantages over ordinary logic in areas like artificial intelligence where a simple true/false statement is insufficient

Traditional Logic

- Propositional logic:
  - Propositional logic is a formal system that uses true statements to form or prove other true statements
  - There are two types of sentences: simple sentences and compound sentences
  - Simple sentences are propositional constants; statements that are either true or false
  - Compound sentences are formed from simpler sentences by using negations $\neg$, conjunctions $\land$, disjunctions $\lor$, implications $\Rightarrow$, reductions $\Leftarrow$, and equivalences $\Leftrightarrow$
- Predicate logic:
  - Onto propositional logic, this adds the ability to quantify variables, so we can manipulate statements about all or some things
  - Two common quantifiers are the existential $\exists$ and universal $\forall$ quantifiers

Formal Fuzzy Logic

- Fuzzy Propositional Logic
  - Like ordinary propositional logic, we introduce propositional variables, truth-functional connectives, and a propositional constant 0
  - Some of these include:
    - Monoidal t-norm-based propositional fuzzy logic
    - Basic propositional fuzzy logic
    - Łukasiewicz fuzzy logic
    - Gödel fuzzy logic
    - Product fuzzy logic
    - Rational Pavelka logic
- Fuzzy Predicate Logic
  - These extend fuzzy propositional logic by adding universal and existential quantifiers in a manner similar to the way that predicate logic is created from propositional logic

Simple Fuzzy Operators

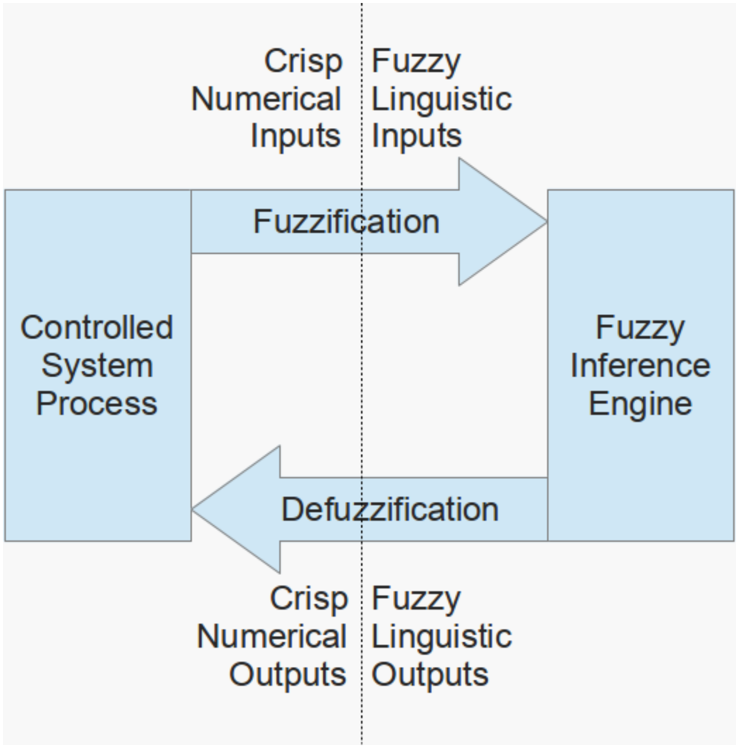
- As described by Zadeh (1973)…
- NOT X = 1 - $\mu_X(y)$
  - e.g. 0.8 cold → (1 – 0.8) = 0.2 NOT cold
- X OR Y (union) = max($\mu_X(y)$, $\mu_Y(y)$)
  - e.g. 0.8 cold, 0.5 rainy → 0.8 cold OR rainy
- X AND Y (intersection) = min($\mu_X(y)$, $\mu_Y(y)$)
  - e.g. 0.9 hot, 0.7 humid → 0.7 hot AND humid

Alternative Interpretations of AND and OR

- Zadeh's definition of AND used the Gödel t-norm, but other definitions are possible using different t-norms
- Common examples:
  - Product t-norm: $\mu_X(y) * \mu_Y(y)$
    e.g. 0.9 hot, 0.7 humid → 0.63 hot AND humid
  - Lukasiewicz t-norm: $\max(\mu_X(y) + \mu_Y(y) - 1, 0)$
    e.g. 0.9 hot, 0.7 humid → 0.6 hot AND humid
- Similar possibilities for OR using corresponding t-conorms:
  - Product t-conorm: $\mu_X(y) + \mu_Y(y) - \mu_X(y) * \mu_Y(y)$
    e.g. 0.8 cold, 0.5 rainy → 0.9 cold OR rainy
  - Lukasiewicz t-conorm: $\min(\mu_X(y) + \mu_Y(y), 1)$
    e.g. 0.8 cold, 0.5 rainy → 1 cold OR rainy

Fuzzy System Overview

- When making inferences, we want to clump the continuous numerical values into sets
- Unlike Boolean logic, fuzzy logic uses fuzzy sets rather than crisp sets to determine the membership of a variable
- This allows values to have a degree of membership with a set, which denotes the extent to which a proposition is true
- The membership function may be triangular, trapezoidal, Gaussian or any other shape

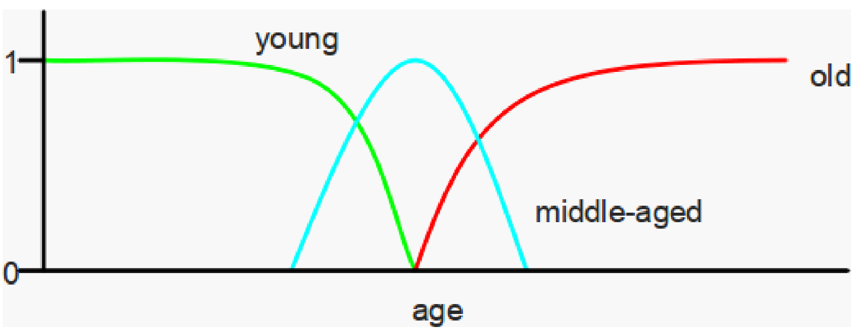


Fuzzification

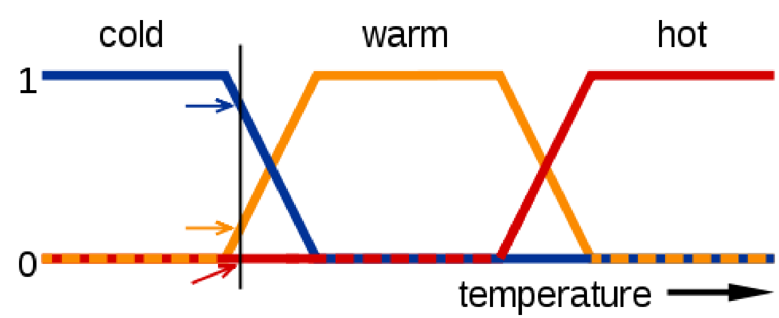
- To apply fuzzy inference, we need our input to be in linguistic values
- These linguistic values are represented by the degree of membership in the fuzzy sets
- The process of translating the measured numerical values into fuzzy linguistic values is called fuzzification
- In other words, fuzzification is where membership functions are applied, and the degree of membership is determined

Membership Functions

- There are largely three types of fuzzifiers:
  - singleton fuzzifier,
  - Gaussian fuzzifier, and
  - trapezoidal or triangular fuzzifier



Gaussian



Trapezoidal

Defuzzification

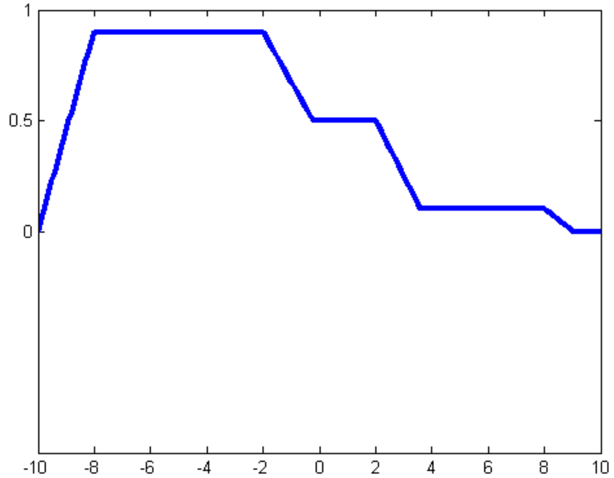
- Defuzzification is the process of producing a quantifiable result in fuzzy logic
- The fuzzy inference will output a fuzzy result, described in terms of degrees of membership of the fuzzy sets
- Defuzzification interprets the membership degrees in the fuzzy sets into a specific action or real-value

Methods of Defuzzification

- There are many methods for defuzzification
- One of the more common types of defuzzification technique is the maximum defuzzification techniques. These select the output with the highest membership function
- They include:
  - First of maximum
  - Middle of maximum
  - Last of maximum
  - Mean of maxima
  - Random choice of maximum

Methods of Defuzzification

- Given the fuzzy output:



- The first of maximum, middle of maximum, and last of maximum would be -2, -5, and -8 respectively as seen in the diagram
- The mean would give the same result as middle unless there is more than one plateau with the maximum value

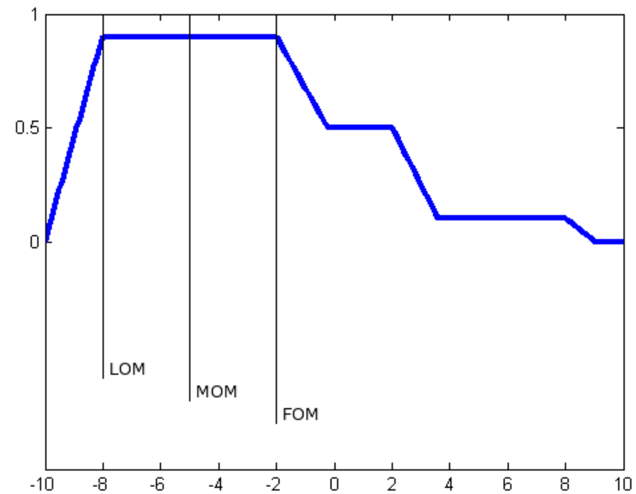


Image Source: http://www.mathworks.com.au/products/fuzzy-logic/examples.html?file=/products/demos/shipping/fuzzy/defuzzdm.html

Methods of Defuzzification

- Two other common methods are:
- Centre of gravity:
  - Calculates the centre of gravity for the area under the curve
- Bisector method:
  - Finds the value where the area on one side of that value is equal to the area on the other side

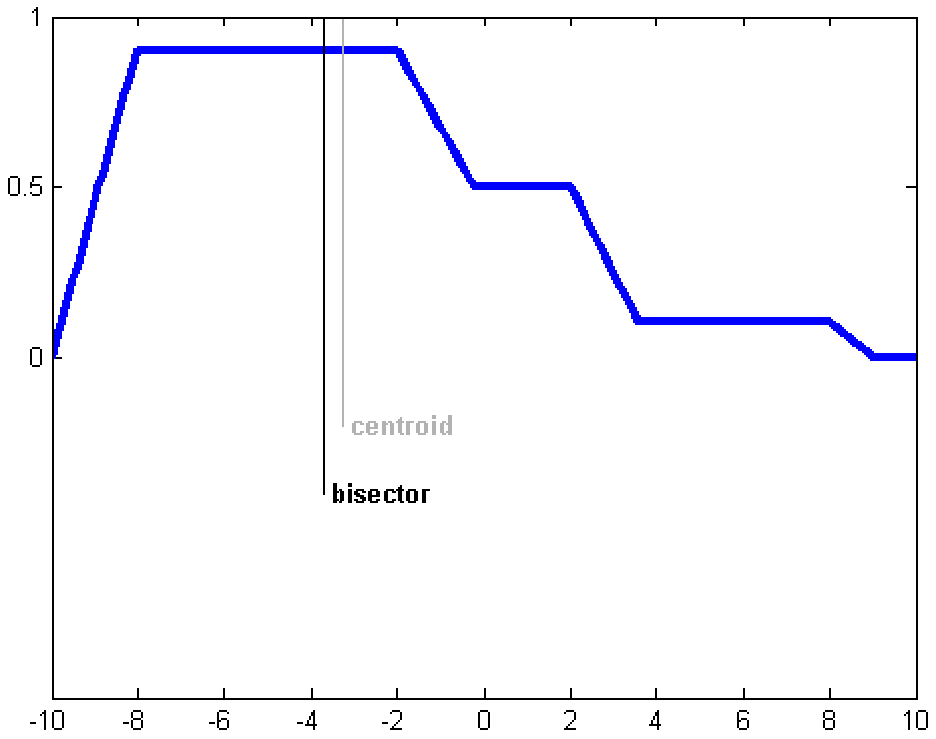


Image Source: http://www.mathworks.com.au/products/fuzzy-logic/examples.html?file=/products/demos/shipping/fuzzy/defuzzdm.html

Fuzzy Control Systems

- A simple example application: an automated cooling fan that can adjust its speed according to the room temperature
- Temperature values are mapped to the fuzzy sets *cold*, *warm* and *hot* based on the following mapping functions for each:

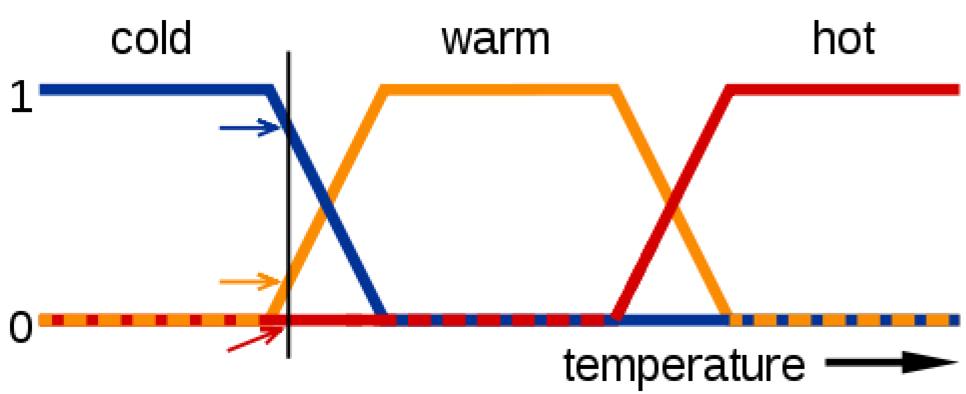


Image: http://en.wikipedia.org/w/index.php?title=File:Fuzzy_logic_temperature_en.svg&page=1

Fuzzy Control Systems

- The inference engine in a fuzzy system consists of linguistic rules
- The linguistic rules consist of two parts:
  - an antecedent block (the conditions), which consists of the linguistic variables
  - a consequent block (the output)

- Fuzzy algorithm
  - Algorithm that includes at least some fuzzy instructions, such as conditional or unconditional action statements
  - Fuzzy conditional statement ($A \longrightarrow B$)
    - Conditional statement in which A and/or B are fuzzy sets
      e.g. IF temperature is *hot* THEN fan speed is *high*
    - Defined in terms of a fuzzy relation between the respective "universes of discourse" of A and B (compositional rule of inference)
      e.g. relation between temperature groupings and fan speeds

A Simple Fuzzy Algorithm Example

- The algorithm used by our cooling fan controller might look like this:

WHILE fan is switched on

IF *cold* THEN stop fan

IF *warm* AND NOT *cooling down* THEN increase fan speed *slightly*

IF *hot* THEN increase fan speed *substantially*

etc…

Types of Fuzzy Algorithms

- Definitional algorithms
  - Define a fuzzy set or calculate grades of membership of elements, e.g.:
    - handwritten characters (what could an "M" look like?)
    - measures of proximity (what counts as *close*?)

- Generational algorithms
  - Generate a fuzzy set
    e.g. an arbitrary sentence in some natural language that needs to be grammatically valid according to various rules

Types of Fuzzy Algorithms (cont'd)

- Relational algorithms
  - Describe a relation between fuzzy variables
  - Can be used to approximately describe behaviour of a system
  - e.g. in our cooling fan example, describing the relation between the input variable (temperature) and output variable (fan speed)

- Decisional algorithms
  - Approximately describe a strategy for performing some task, e.g.:
    - approaching a set of traffic lights (should we slow down, stop or proceed at current speed?)
    - navigating a robot towards a goal while avoiding obstacles

Applications of Fuzzy Logic

- Control Systems
  - Consumer systems
    - automatic transmissions
    - washing machines
    - camera autofocus
  - Industrial systems
    - aircraft engines
    - power supply regulation
    - steam turbine start-up

A commercial tool for building embedded fuzzy systems

Image source: http://www.bytecraft.com/Fuzz-C__Fuzzy_Logic_Preprocessor

Applications of Fuzzy Logic

- Artificial Intelligence
  - Robot motion planning
  - Image segmentation
  - Medical diagnosis systems

Fuzzy Logic Control Systems

Why use fuzzy logic for control?

- Simple systems:
  - Low development costs
  - Low maintenance costs
- Complex systems:
  - Reduced run-time
  - Reduced search space for efficient optimisation

How can fuzzy logic achieve this?

Fuzzy Control System Development

Fuzzy logic:

- Is used to quickly translate from expert knowledge to code
- Expert knowledge reduces the search space when optimising the system

Fuzzy Control System Development

1. Identify performance measure
2. Select input/output variables
3. Determine fuzzy rules
   - Talking to an expert
   - Data mining
4. Decide on membership functions for the fuzzy variables
5. Tune membership functions and/or rules

Aircraft Engine Control

Engine:

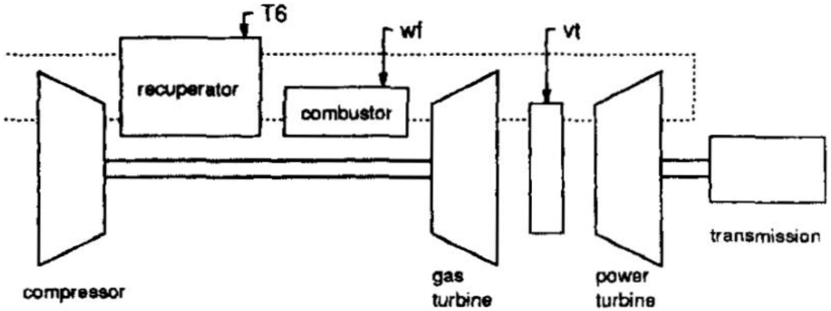
- General Electric's LV100 Turboshaft engine

Existing solution:

- 10 low level controllers
- Only one controller runs at a time
- The controller is selected based on current conditions (a crisp mode selector)

Problem:

- Abrupt changes when switching controllers
- Poor fuel economy
- High peak operating temperature



Schematic of the LV100 engine.

An AGT1500 Turboshaft engine being installed into a tank. The AGT1500 is the predecessor to the LV100.

Top Image Source: Bonissone et al. 1995
Bottom Image Source: http://en.wikipedia.org/wiki/Honeywell_AGT1500

Engine Control: Solution

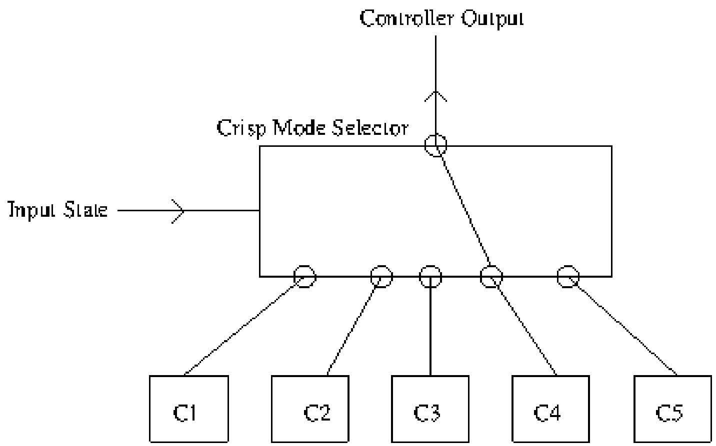
Replace the crisp mode selector with:

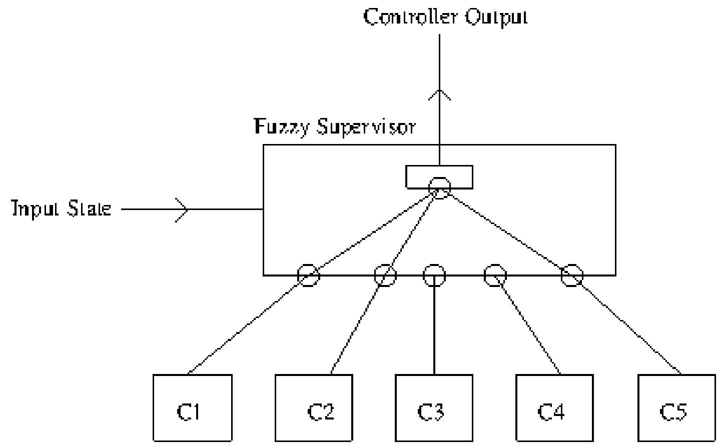
A **Fuzzy** Supervisor

- 24 operation modes → 24 operation rules
- Blend outputs of existing controllers

Result (from simulation):

- Significantly reduced fuel consumption
- Did not achieve desired reduction in operating temperature





Engine Control: Solution 2

Hierarchical fuzzy control system

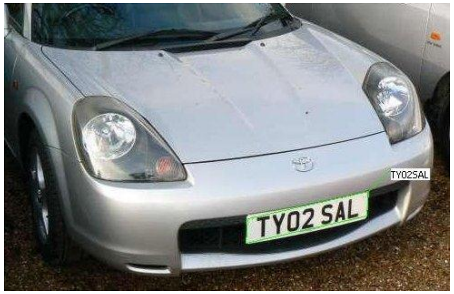
- Use a fuzzy supervisor
- Replace the low level controllers with fuzzy controllers
  - 6 fuzzy controllers used, 49 rules each
  - Controllers govern: fuel flow, turbine nozzle area
- Fuzzy controllers act on small time scales
- Fuzzy supervisor acts on large time scales

Engine Control: Results

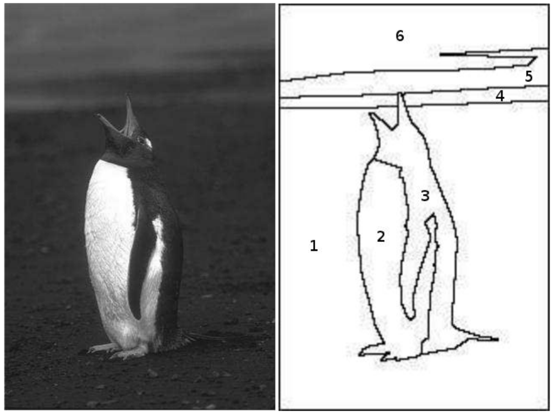
- Reduced fuel consumption
- Lower maximum temperature
  - Increased component life
- Improved performance
- Development time was ¼ of the time taken to develop the existing control system

Image Segmentation

- Decompose an image into regions
- Regions have similar properties
  - Colour, Texture
- Can be a clustering problem or a classification problem



An example of licence plate segmentation.

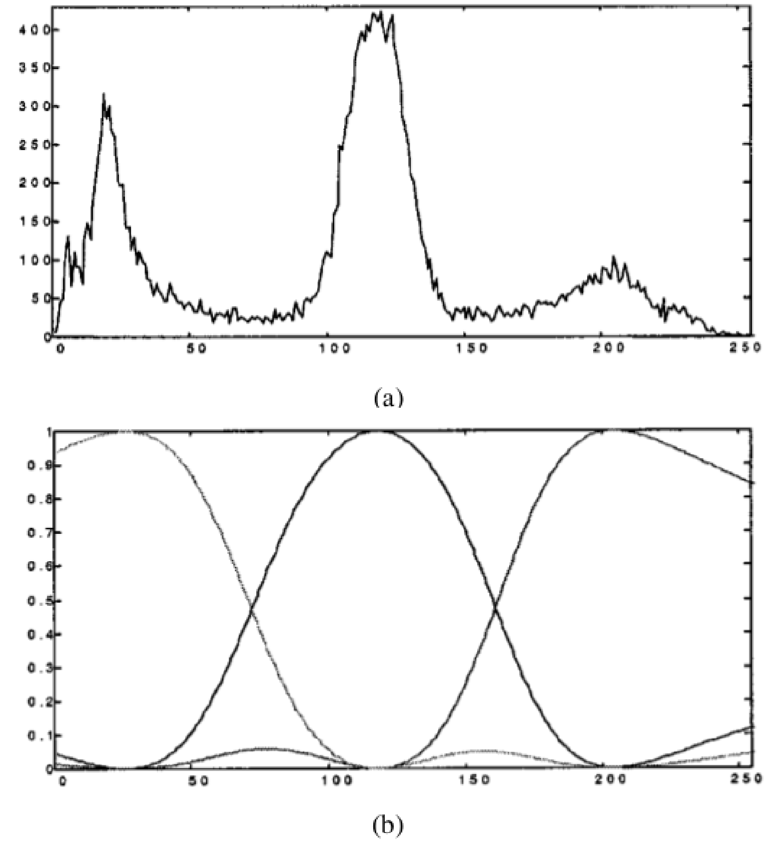


An example of image segmentation. The input image is on the left. The output image is on the right.

Top Image Source: http://en.wikipedia.org/wiki/Automatic_number_plate_recognition
Bottom Image Source: www.cs.toronto.edu/~jepson/csc2503/segmentation.pdf

Neuro-Fuzzy Image Segmentation

1. Decide on the number of fuzzy variables (number of regions)
2. Calculate the grey-scale histogram
3. Apply fuzzy c-Means (FCM) to get membership functions

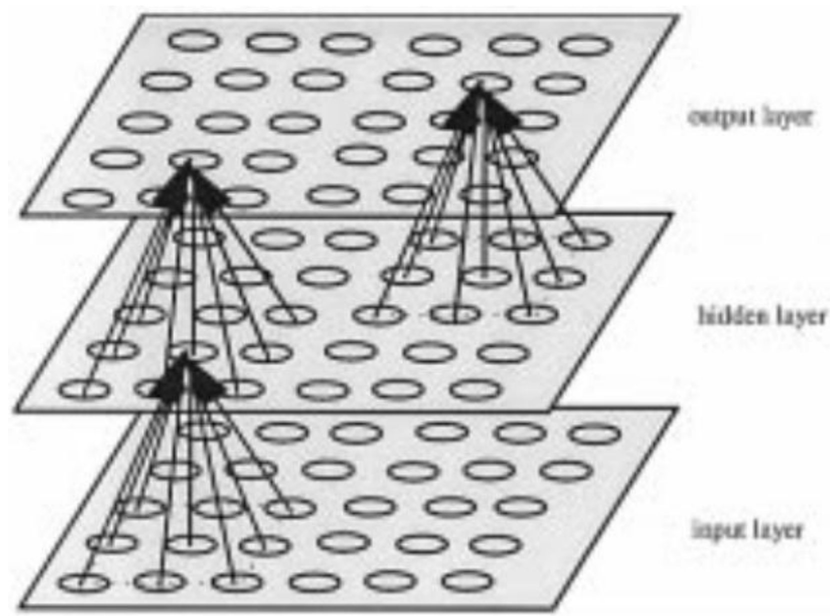


a) a grey scale histogram
b) three fuzzy membership functions for (a)

Image Source: Boskovitz & Guterman 2002

Neuro-Fuzzy Image Segmentation

4. Pass the MxN image through an LP with layers of size MxN
5. Fuzzify the pixels in the output layer

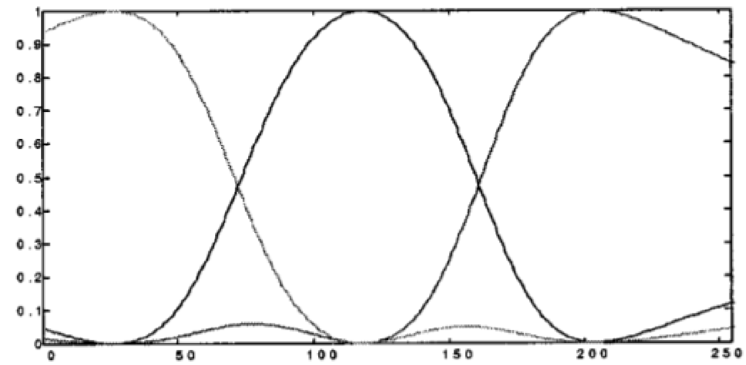


The structure of the neural network. Circles are neurons. Arrows are connections between neurons.

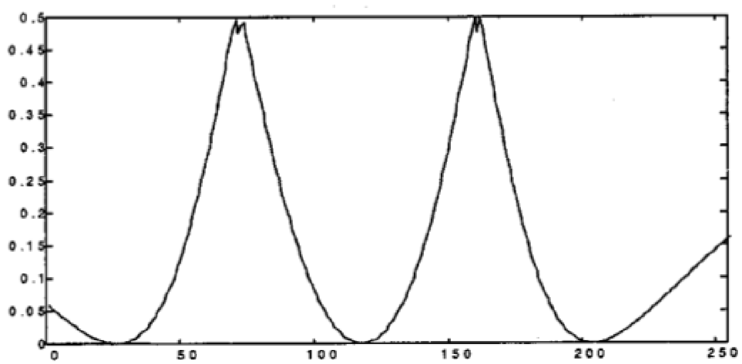
Image Source: Boskovitz & Guterman 2002

Neuro-Fuzzy Image Segmentation

6. Calculate the total fuzziness (entropy) of the output
7. Using fuzzy entropy as the error function, train the MLP with back-propagation



(b)



(d)

b) three fuzzy membership functions for histogram (a)
d) the fuzziness of each possible grey-scale value

Image Source: Boskovitz & Guterman 2002

Neuro-Fuzzy Image Segmentation

Input panda image

Output panda image

Image Source: Boskovitz & Guterman 2002

References – General

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