Artificial Intelligence

Topic 12

Logical Inference

Reading: Russell and Norvig, Chapter 7, Section 5

Outline

- ♢ Inference systems
- ♢ Soundness and Completeness
- ♢ Proof methods
  - normal forms
  - forward chaining
  - backward chaining
  - resolution

From Entailment to Inference

- We have answered what it means to say α *follows from* from a knowledge base KB:
  - $KB \models \alpha$
- We have seen that this can be determined semantically by *model checking* or by *truth table enumeration*
  - 2^n models or rows for n symbols
- Is there a better way?
  - can we do it from syntax alone?
  - can we automate it?
  - can we even turn it into a programming language?

Inference Systems

Inference system - set of *rules* for deriving new sentences from existing ones

- AKA Proof System, Derivation System, Theorem-Proving System
- rules operate directly on syntax

Example:

$P_1 =$ Socrates is a man
$P_2 =$ Socrates is mortal
$P_1 \Rightarrow P_2$ (If Socrates is a man, then Socrates is mortal)

Assume $KB = \{P_1, P_1 \Rightarrow P_2\}$.

We know $KB \models P_2$. (check)

What about inference rules?

Inference Systems

Modus Ponens

$$\frac{\alpha \qquad \alpha \Rightarrow \beta}{\beta}$$

pattern matching — from sentences that match α and $\alpha \Rightarrow \beta$, generate a new sentence that matches β

More examples...

And Elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Or Introduction

$$\frac{\alpha}{?}$$

Inference Systems

Modus Tolens

$$\frac{\neg\beta \qquad \alpha \Rightarrow \beta}{\neg\alpha}$$

An inference system may contain one or more inference rules.

Notation:

$KB \vdash \alpha =$ sentence α can be derived from KB using the rules of the inference system

Soundness and Completeness

In fact we could make up any inference rule we like. How about:

And Introduction

$$\frac{\alpha}{\alpha \wedge \beta}?$$

Why wouldn't we want this rule in our inference system?

Soundness and Completeness

We only want rules that "correspond" to logical consequences. Formally...

Soundness: an inference system is *sound* if
whenever $KB \vdash \alpha$, it is also true that $KB \models \alpha$

That is, it *only* allows you to generate logical consequences.

Completeness: an inference system is *complete* if
whenever $KB \models \alpha$, it is also true that $KB \vdash \alpha$

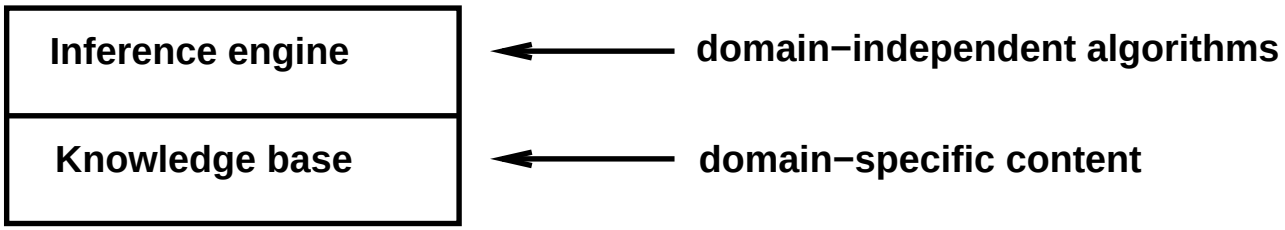
That is, it allows you to generate *all* logical consequences.

(Which do you think is worse, sound but not complete, or complete but not sound?)

Ideally we would like to use an inference system that is both sound and complete.

Review of Progress

Recall our logical agent:



- Knowledge base = set of sentences in a **formal** language ✔
- TELL it what it needs to know ✔
  $(KB \leftarrow KB \cup \{\alpha\})$
- ASK — answers should follow from the KB ✔?
  $(KB \vdash \alpha)$

A sound and complete inference system means that if α follows from KB then there is a sequence of rule applications that allow you to generate α starting with KB — but it doesn't tell you *how* to get there!

Proof Methods

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it

Proof Methods

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications

  Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Examples

- forward and backward chaining (Horn form)
- resolution (conjunctive normal form)

Horn Form

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- ♢ *proposition symbol* or
- ♢ *(conjunction of symbols* $\Rightarrow$ *symbol)*

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

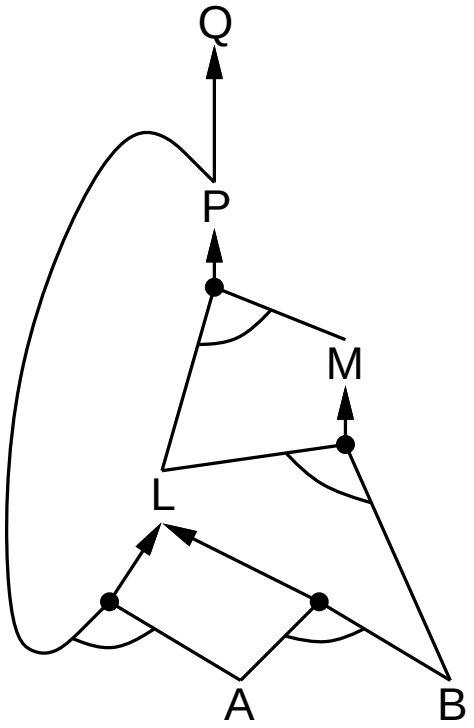
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in linear time.

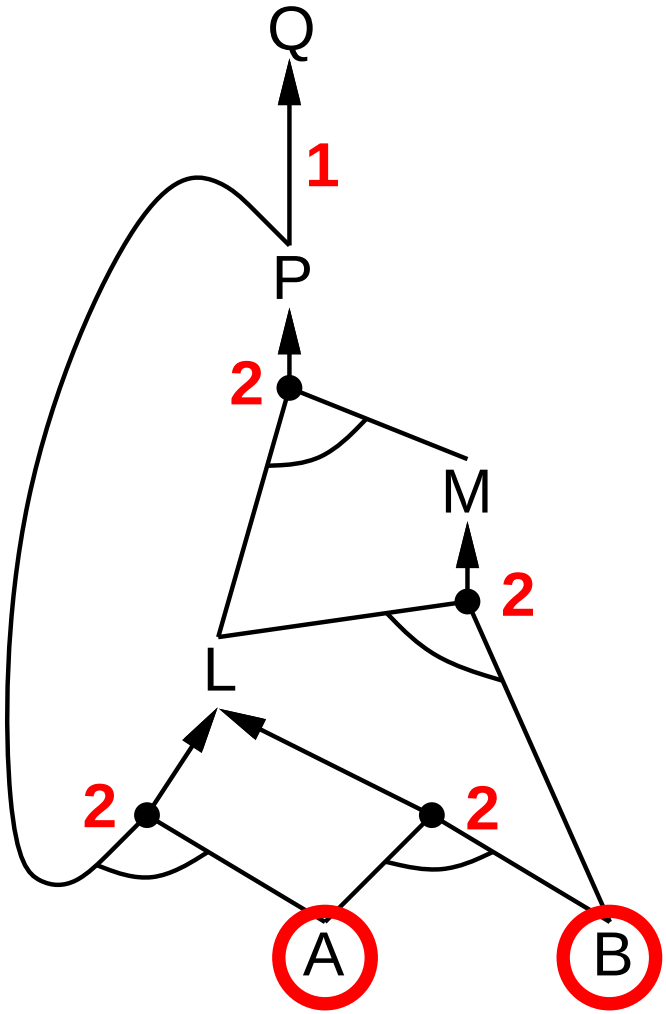
Forward chaining

Idea: systematically iterate through knowledge base,
fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

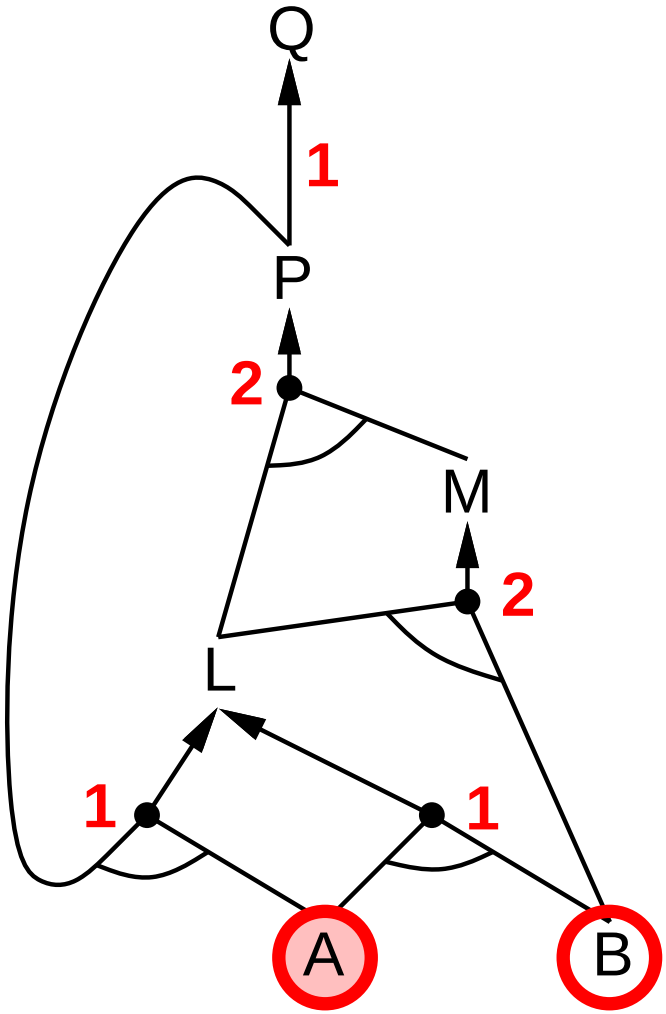
$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining example



Forward chaining example



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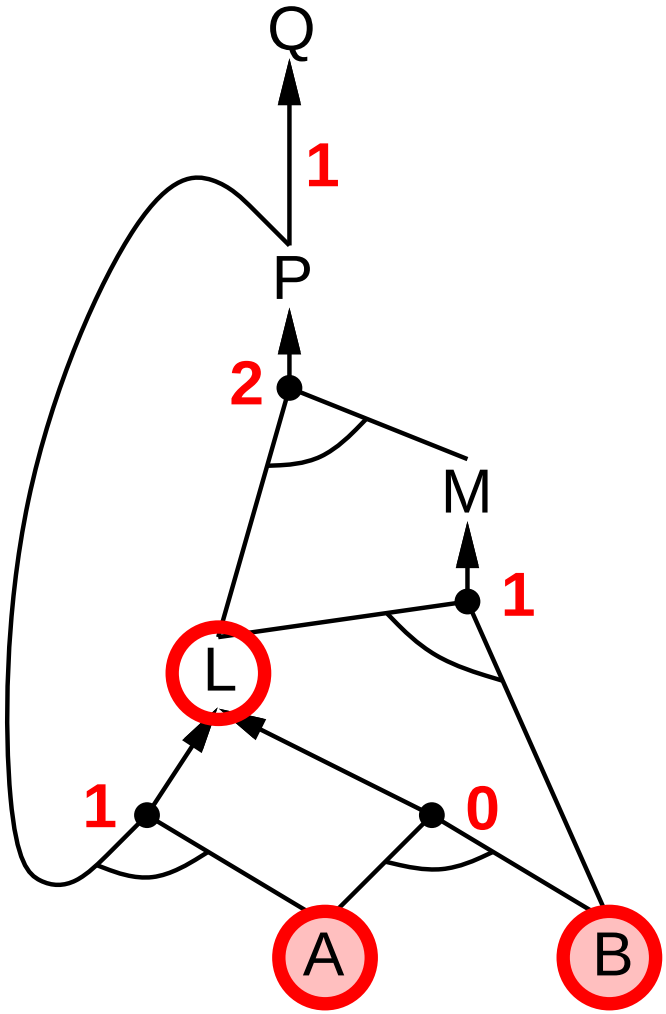

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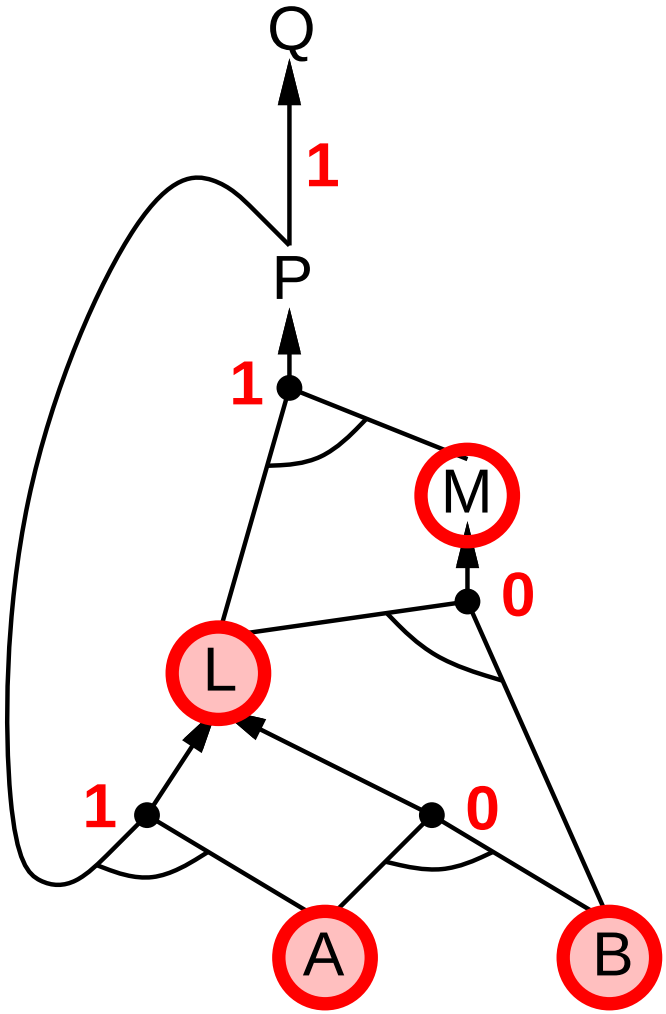

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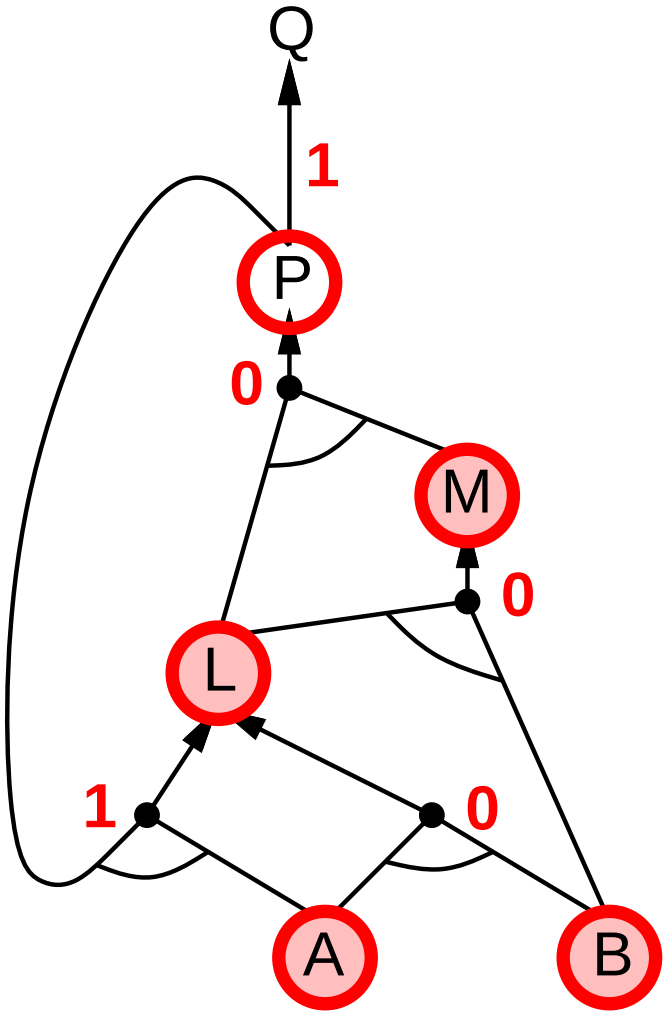

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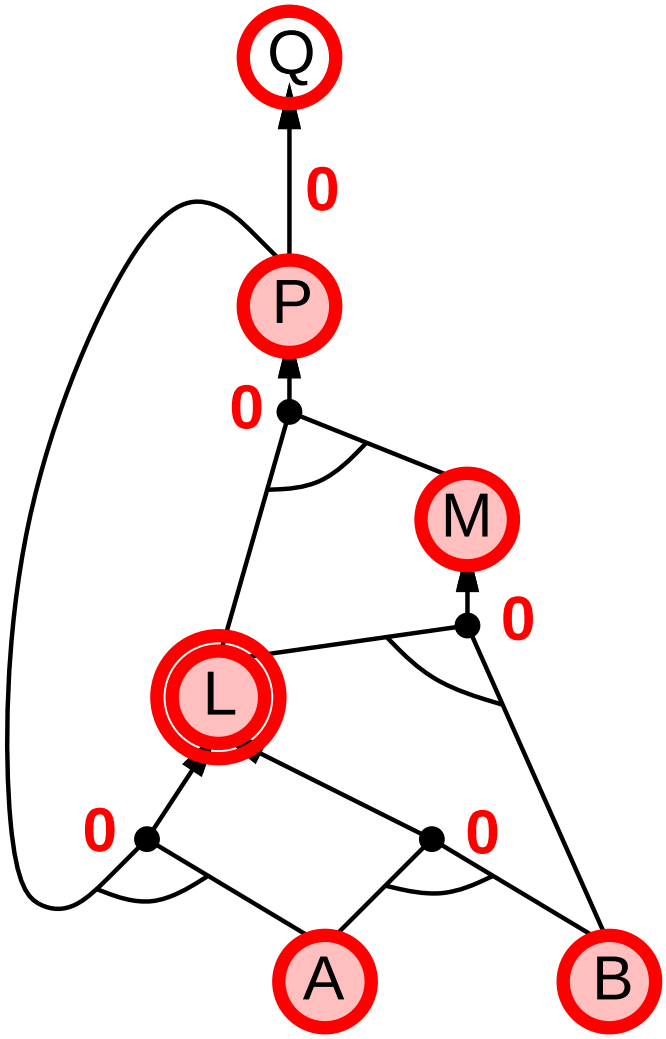

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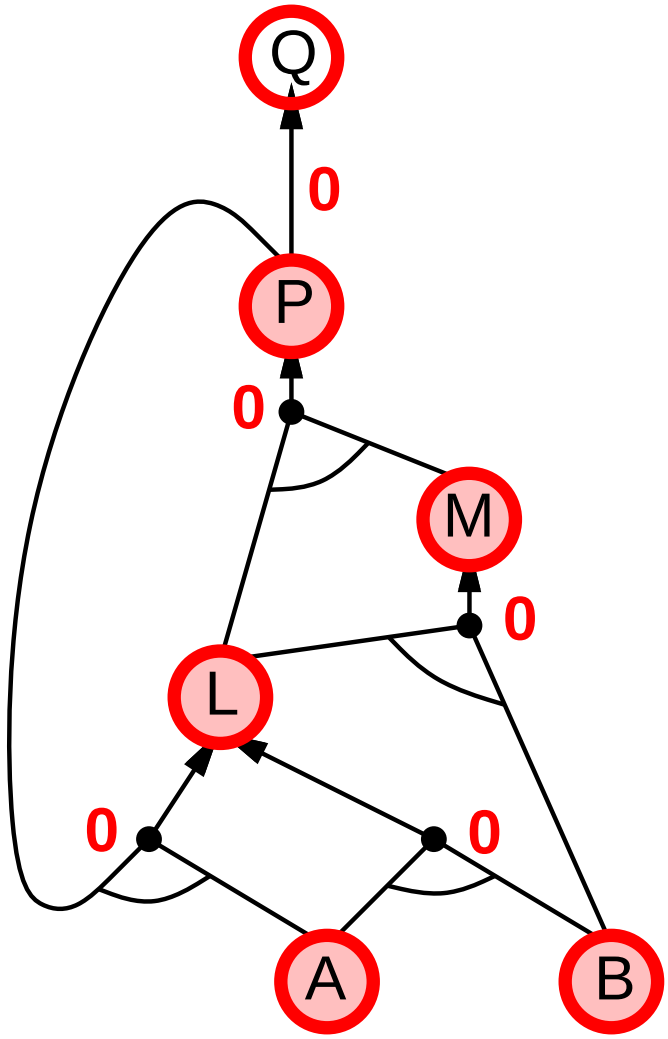

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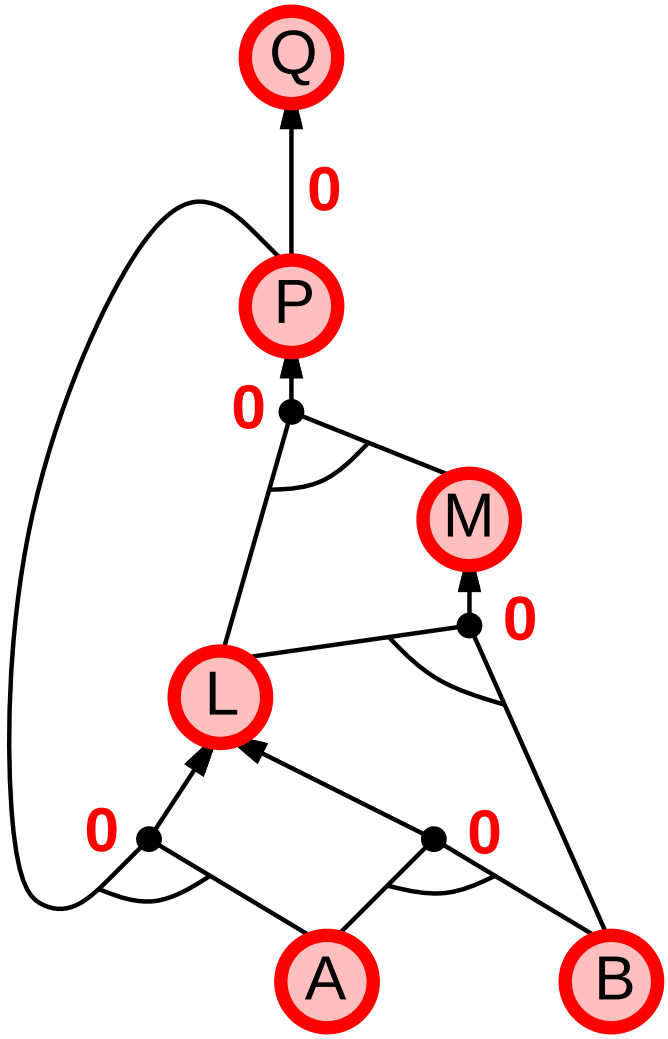


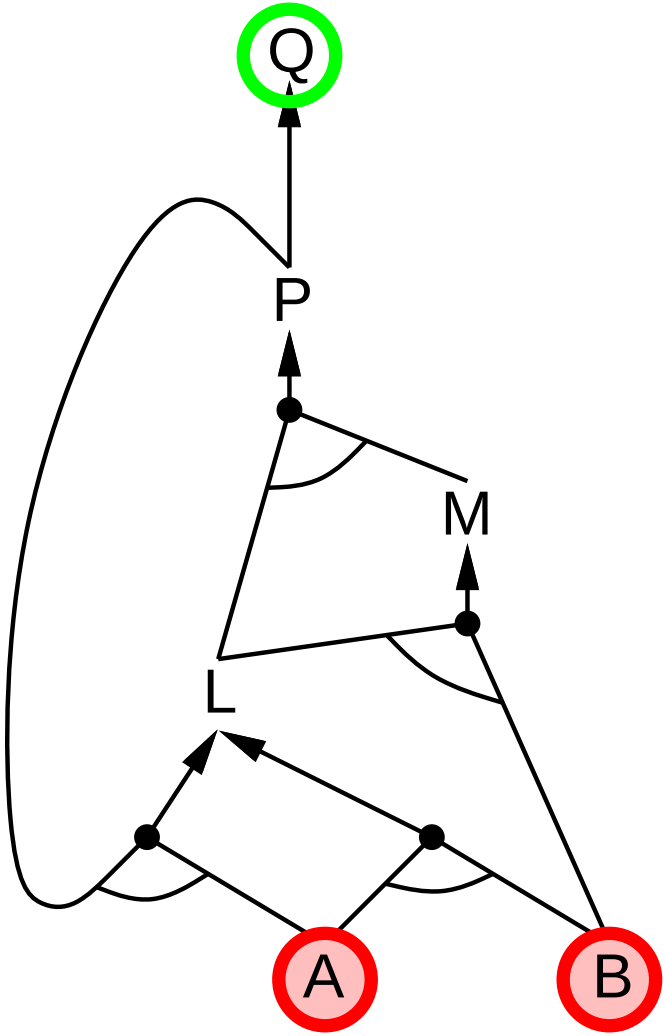
Backward chaining

Idea: work backwards from the query q:
to prove q by BC,
check if q is known already, or
prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
1) has already been proved true, or
2) has already failed

Backward chaining example



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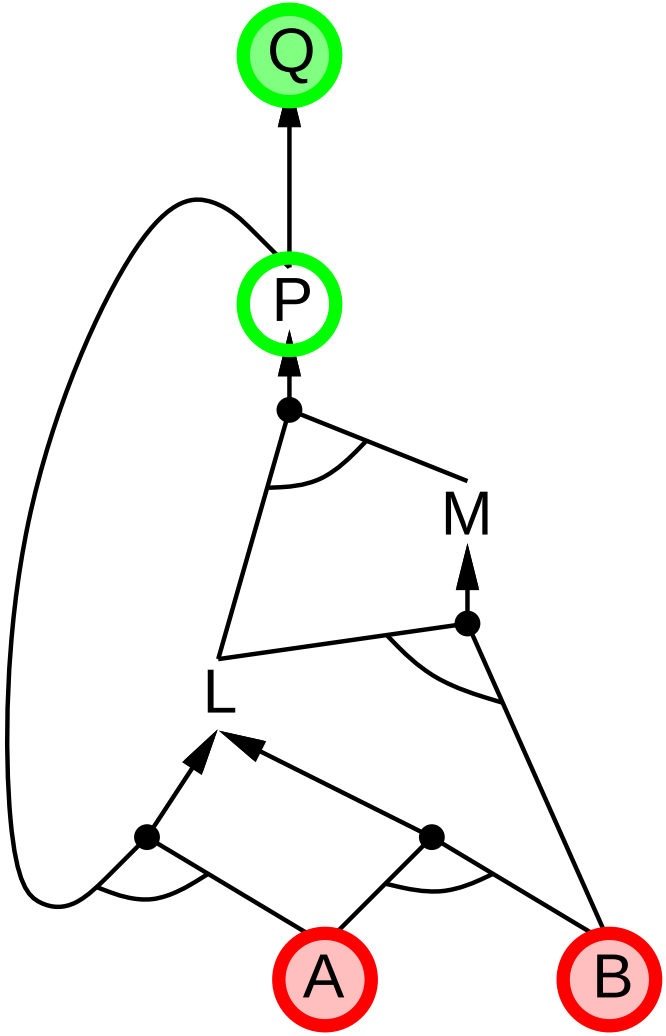

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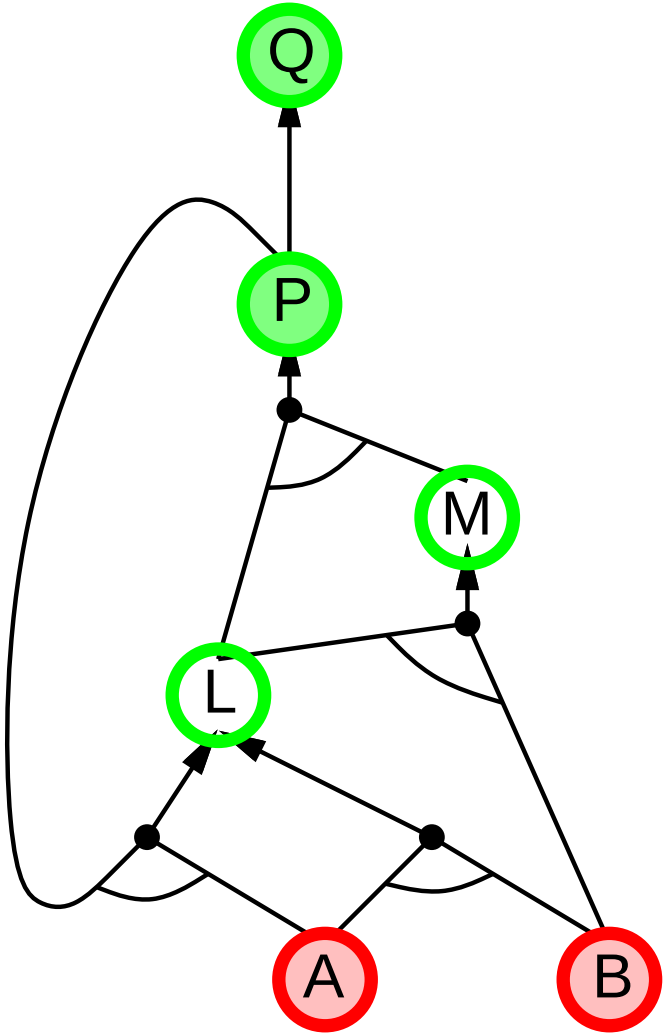

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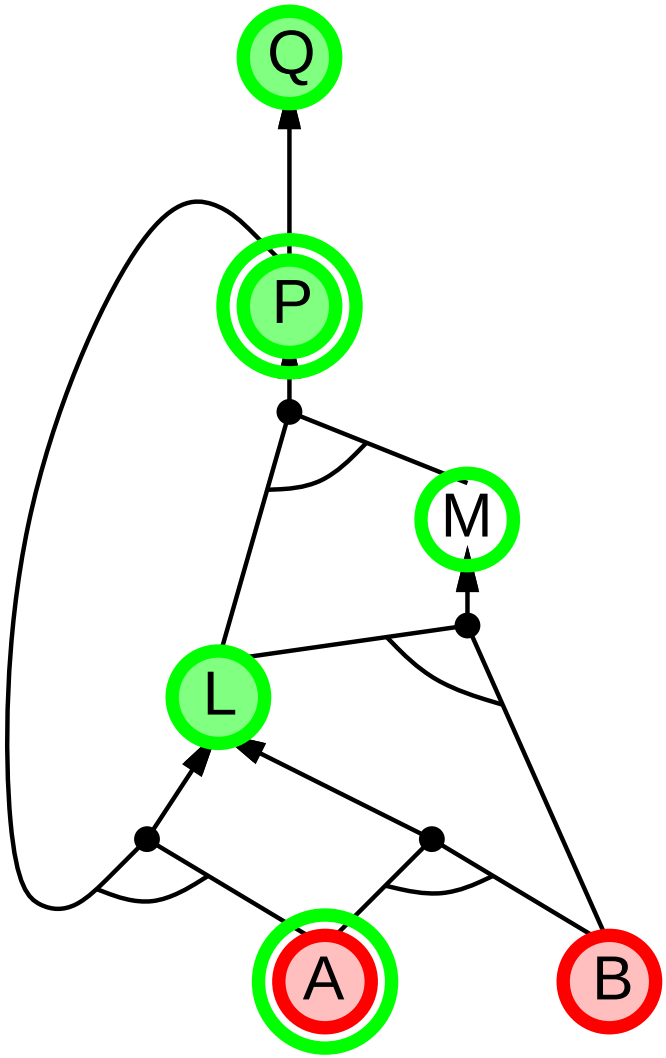

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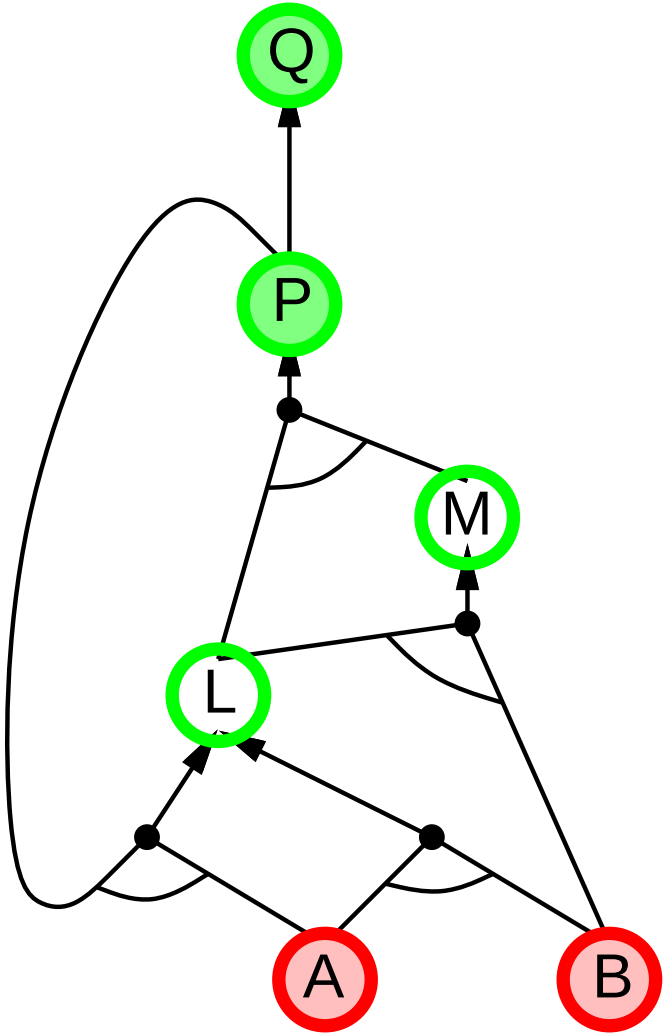

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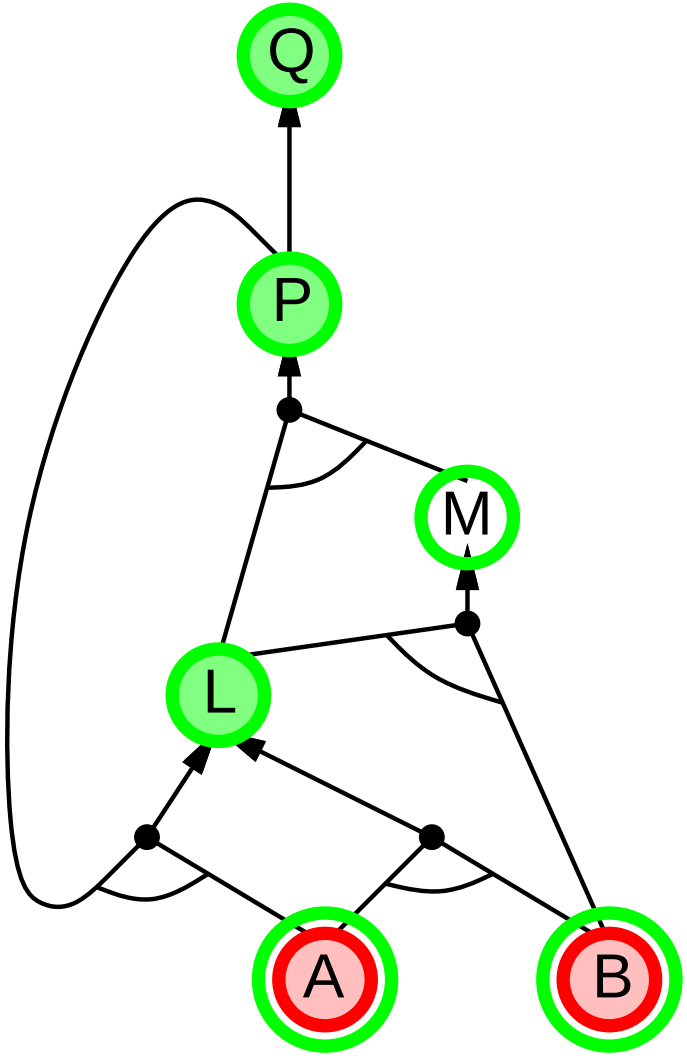

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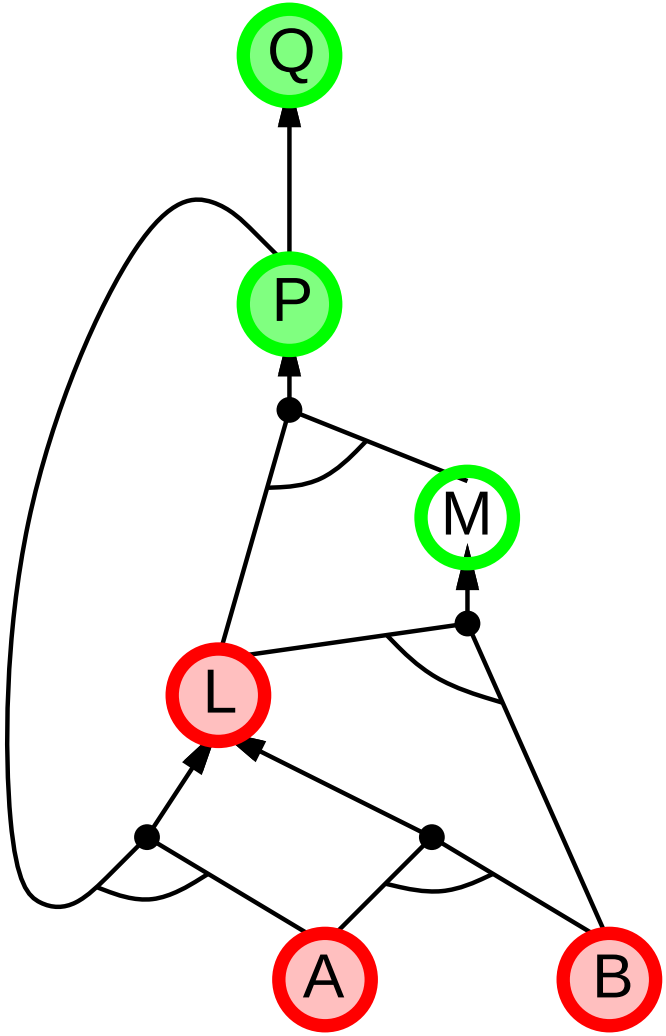

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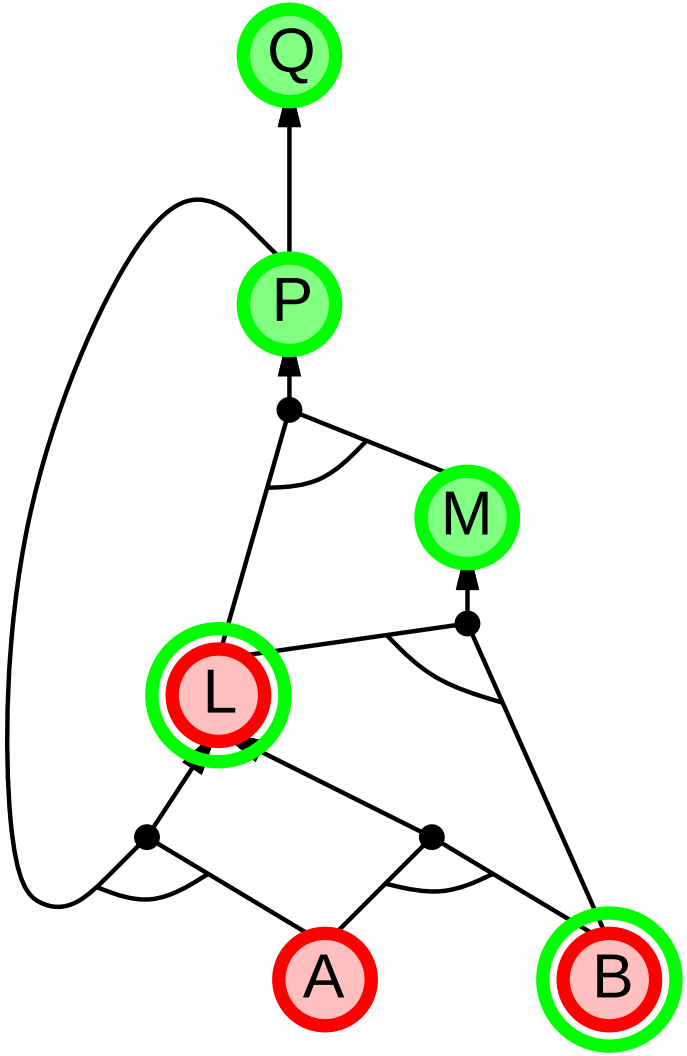

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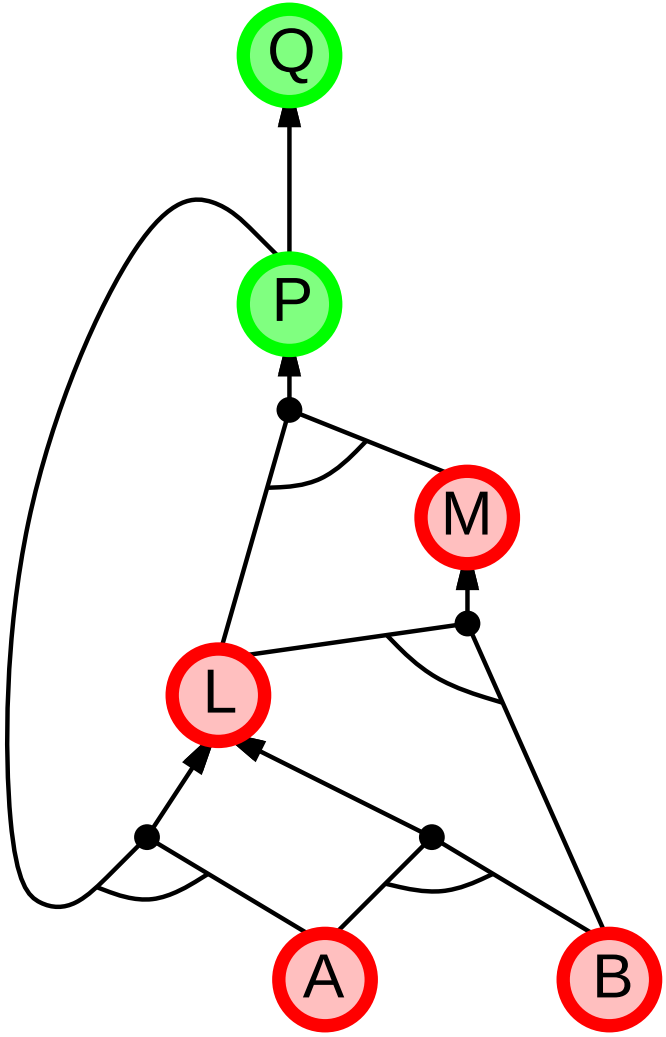

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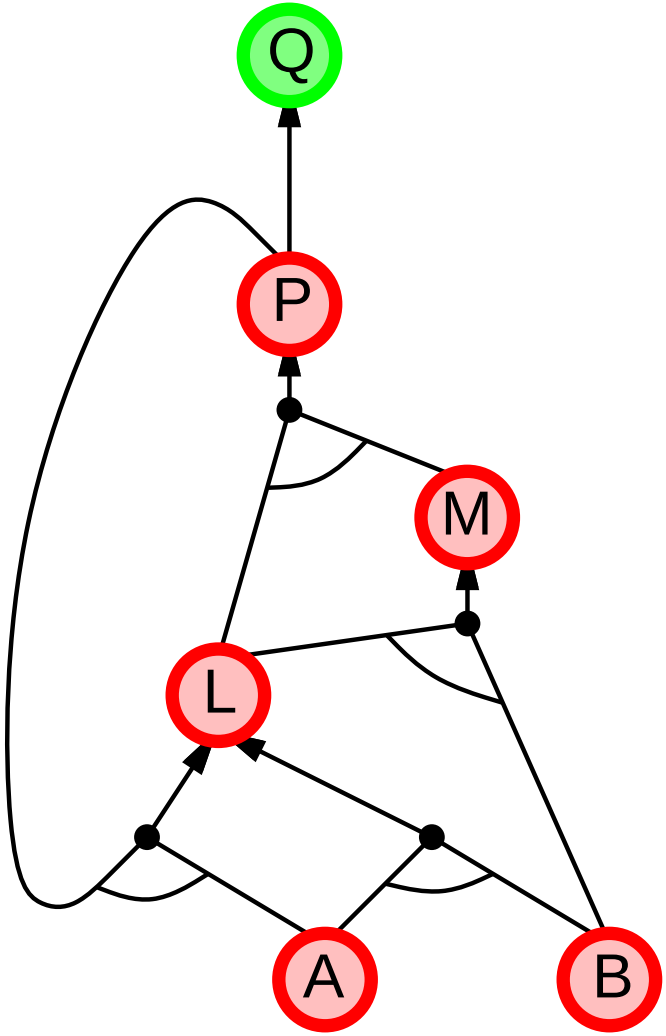

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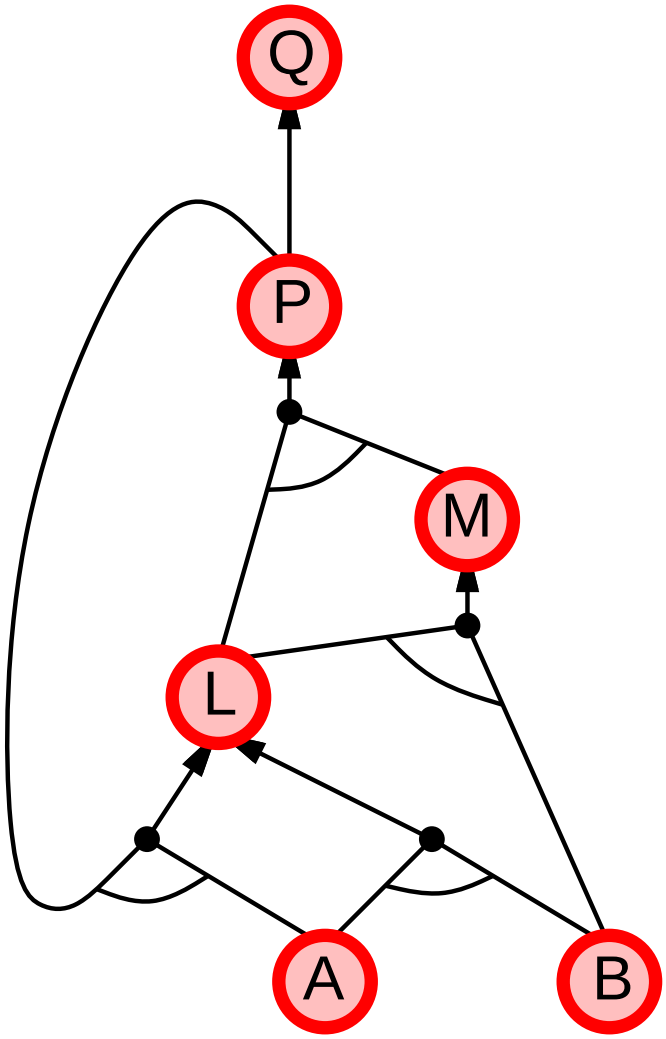


Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)

conjunction of $\underbrace{\textbf{disjunctions of literals}}_{\textbf{clauses}}$

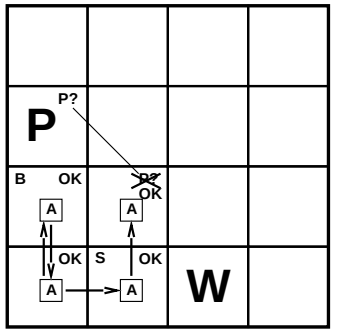
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution algorithm

"Proof by contradiction"

Based on the fact that $KB \models \alpha$ iff $KB \cup \{\neg\alpha\}$ is unsatisfiable (prove!)

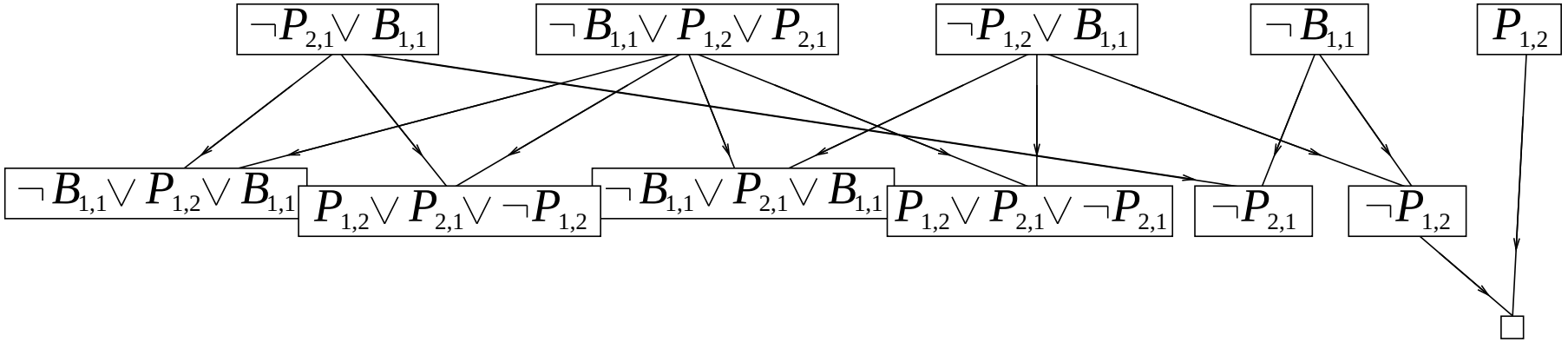
Unsatisfiability in CNF is indicated by the empty clause

We repeatedly apply the resolution rule to $\neg\alpha$ and its consequences until we derive the empty clause.

Exercise: What is the complexity of the conversion to CNF?
What is the complexity of the resolution algorithm?

Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \qquad \alpha = \neg P_{1,2}$



Resolution combined with first-order logic is the key mechanism used in logic programming (eg PROLOG).

Efficient Reasoning Algorithms

The satisfiability problem for propositional logic in both infeasible (NP-complete) and very useful (TSP, CSP, planning, etc).

So if an agent is *required* to perform propositional reasoning, what kind of efficient mechanisms are available?

We will look at two possible algorithms: $\diamondsuit$ The *Davis-Putnam algorithm* is a recursive DFS for satisfying models of a formula, aided by some heuristics; and $\diamondsuit$ *WALKSAT* is a randomized algorithm that performs a local search for a satisfying model.

The DPLL Algorithm (1962)

DPLL is a variation fo the Davis-Putnam algorithm. It takes the input as a senetnce in CNF, and iterates through potential models using the following heuristics:

- **Early Termination** The algorithm recognizes if a clause is true (one of its literals is true) or if a sentence of false (one of its clauses is false) and therefore does not need to search redundant branches of the search tree.
- **Pure Symbols** A *pure symbol* is a symbol that has the same sign in all (active) clauses. These symbls can be ignored.
- **Unit Clause** A *unit clause* is a clause with just one (active) literal. A unit clause dictates the value of that literal in all the other clauses.

function DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

inputs: *s*, a sentence in propositional logic

clauses $\leftarrow$ the set of clauses in the CNF representation of s

symbols $\leftarrow$ a list of the proposition symbols in *s*

return DPLL(*clauses*, *symbols*, [])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

if every clause in *clauses* is true in *model* **then return** *true*

if some clause in *clauses* is false in *model* **then return** *false*

P, *value* $\leftarrow$ FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, $[P = value|model]$)

P, *value* $\leftarrow$ FIND-UNIT-CLAUSE(*clauses*, *model*)

if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, $[P = value|model]$)

P $\leftarrow$ FIRST(*symbols*); *rest* $\leftarrow$ REST(*symbols*)

return DPLL(*clauses*, *rest*, $[P = true|model]$) **or** DPLL(*clauses*, *rest*, $[P = false|model]$)

The WALKSAT algorithm

The *WALKSAT* algorithm simply performs a random walk over all models hoping to find a model that satisfies a sentence in CNF.

For each step of the walk it *flips* the value of a symbol (proposition) and tests if the sentence becomes true.

The algorithm nondeterministically chooses either a randomly selected proposition to flip, or chooses the proposition that maximizes the number of satisfied clauses.

The WALKSAT algorithm

function WALKSAT(*clauses*, *p*, *max-flips*) **returns** a satisfying model or *failure*

inputs: *clauses*, a set of clauses in propositional logic

p, the probability of choosing to do a "random walk" move, typically around 0.5

max-flips, number of flips allowed before giving up

model ← a random assignment of *true*/*false* to the symbols in *clauses*

for $i = 1$ **to** *max-flips* **do**

if *model* satisfies *clauses* **then return** *model*

clause ← a randomly selected clause from *clauses* that is false in *model*

with probability *p* flip the value in *model* of a randomly selected symbol from *clause*

else flip whichever symbol in *clause* maximizes the number of satisfied clauses

return *failure*

More Logics

Although propositional logic is computationally attractive, it lacks expressive power in practice.

eg. How would you say "All men are mortal" or "All squares adjacent to a pit have a breeze"?

There are many other logics that extend propositional logic, eg:

- first-order logic introduces objects, functions, relations, variables, quantifiers (*for all*, *there exists*)
- higher order logics allow the logic to refer to its own constructs
- temporal logics introduce specific structures to represent time steps
- modal logics introduce possibility and necessity
- probabilistic logics introduce the probability a statement is true
- fuzzy logics introduce a degree of membership to a class

Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Forward, backward chaining are linear-time, complete for Horn clauses
Resolution is complete for propositional logic

Resolution is the basis of the Prolog programming language
Uses first-order logic — more expressive power