Informed Search Algorithms

CITS3001 Algorithms, Agents and Artificial Intelligence

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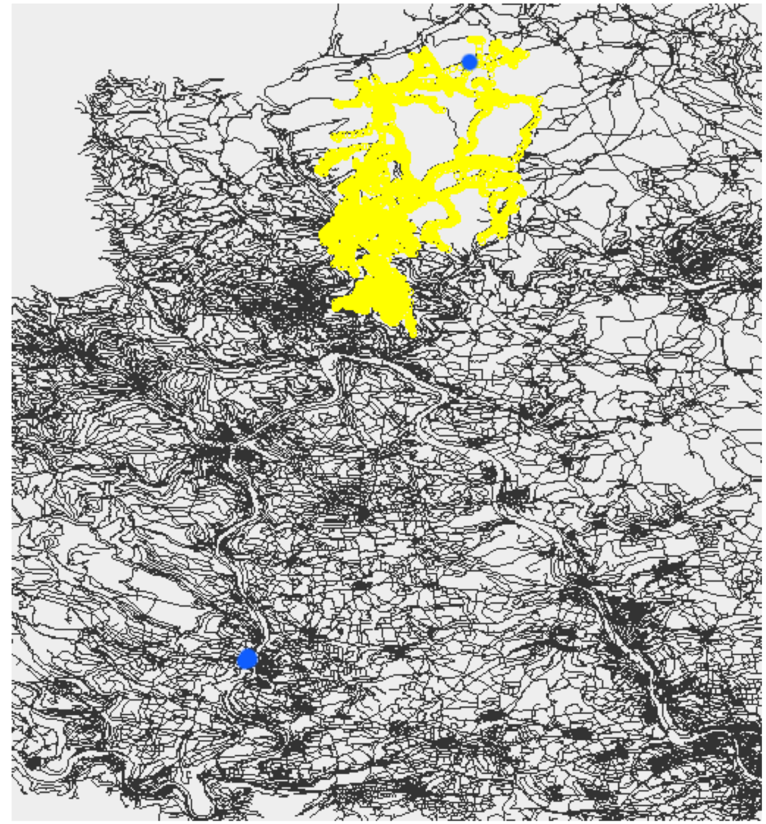
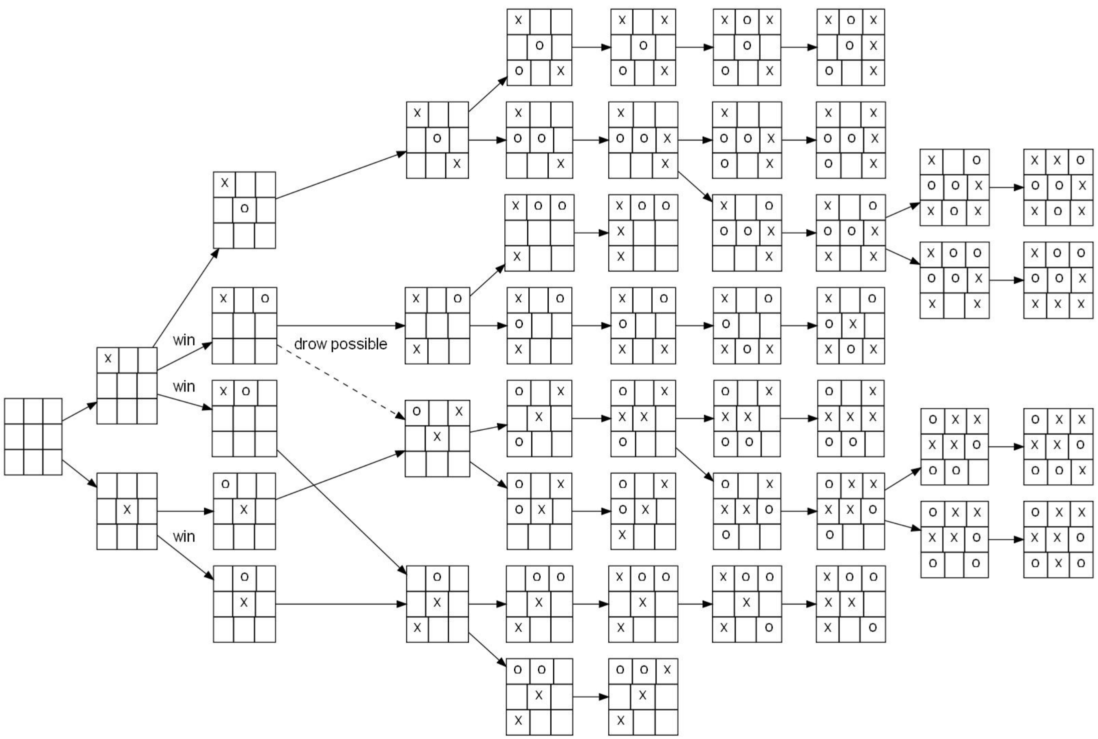
Introduction

- We will introduce informed search algorithms
- We will discuss the *A** algorithm
  - Its proof of optimality
  - Heuristics for improving its performance
  - Memory-bounded versions of *A**





Uniformed vs Informed Search

- Recall uninformed search
  - Selects nodes for expansion on the basis of distance/cost from the start state
    - e.g. which level in the tree is the node?
  - Uses only information contained in the graph
    (i.e. in the problem definition)
  - No indication of *distance to go*

- Informed search
  - Selects nodes for expansion on the basis of some *estimate* of distance to the goal state
  - Requires additional information:
    - *heuristic rules*, or
    - *evaluation function*
  - Selects "best" node, i.e. most promising

- Examples
  - Greedy search
  - *A**

Greedy search of Romania

- To the map of Romanian roads, add the straight-line distance from each city to the goal
- These straight-line distances are *estimates* of how far is left to go.

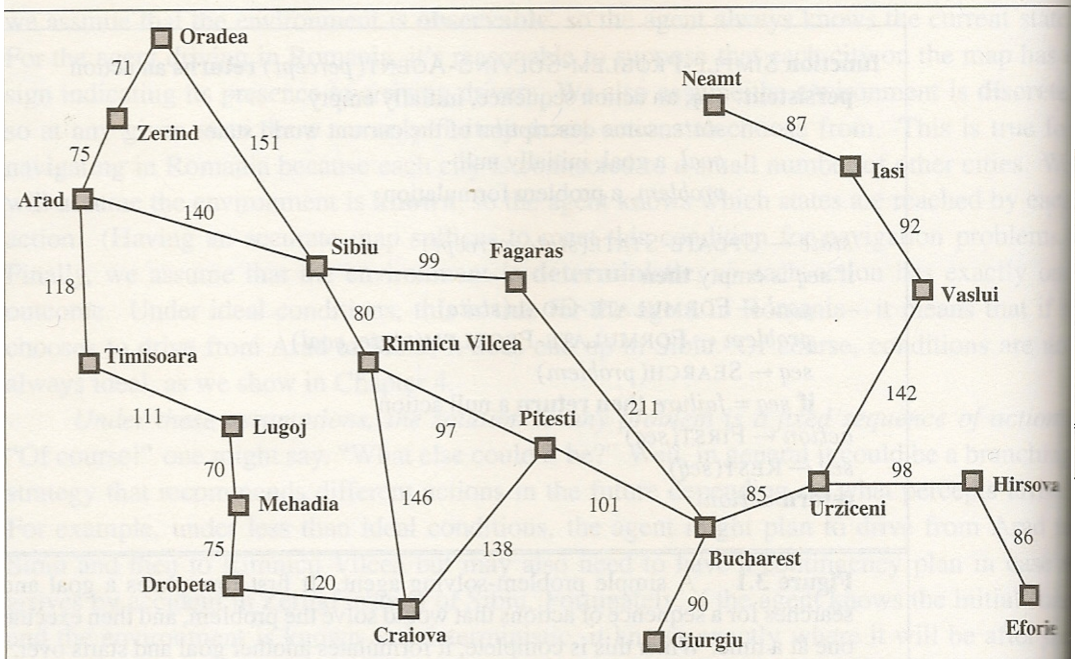


Figure 3.2 A simplified road map of part of Romania.

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 3.22 Values of h_{SLD}—straight-line distances to Bucharest.

Greedy Search

- Greedy search always selects the unvisited node with the smallest estimate
  - The one that *appears to be* closest to the goal
- The evaluation function or heuristic *h(n)* here is the estimate of the cost of getting from *n* to the goal
  - $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

- Complete: not always
- Optimal: no, returns first goal found
- Time: $O(b^m)$ (worst case), but highly dependent on the heuristic's performance
- Space: $O(b^m)$, keeps all nodes in memory

- The complexities are expressed in terms of
  - *b*: maximum *branching factor* of the tree
  - *m*: maximum *depth* of the search space
  - *d*: depth of the *least-cost* solution

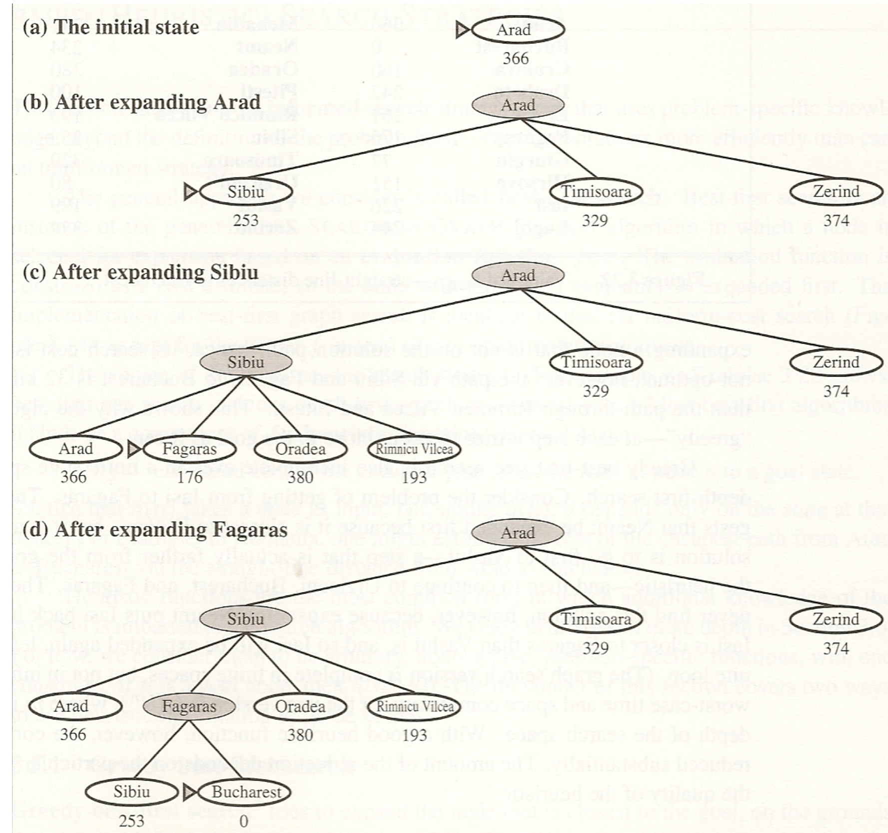


Figure 3.23 Stages in a greedy best-first search for Bucharest with the straight-line distance heuristic h_{SLD}. Nodes are labeled with their h-values.

A* search

- Greedy search minimises estimated path-cost to goal
  - But it's neither optimal nor even always complete
- Uniform-cost search minimises path-cost from the start
  - Complete and optimal, but expensive
- Can we get the best of both worlds?

- Yes – use *estimate of total path-cost* as our heuristic

- *f(n) = g(n) + h(n)*
  - *g(n) =* ***actual*** *cost from start to n*
  - *h(n) =* ***estimated*** *cost from n to goal*
  - *f(n) = estimated total cost from start to goal* ***via n***
-
- Hence *A**
  - There were a series of algorithms, *A1*, *A2*, *etc*. that combined to make *A**

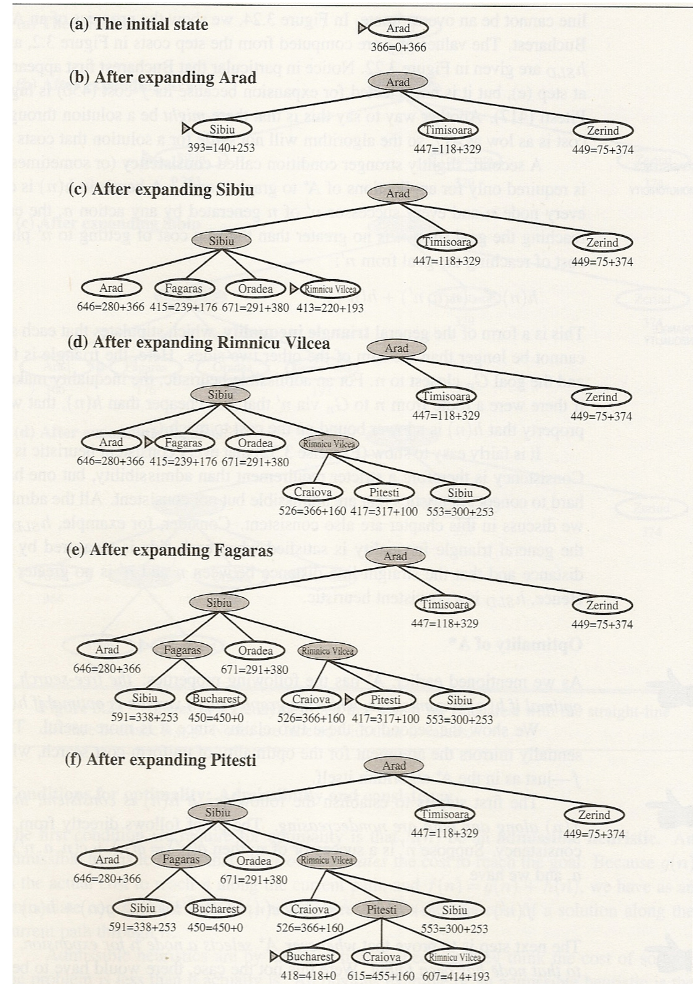
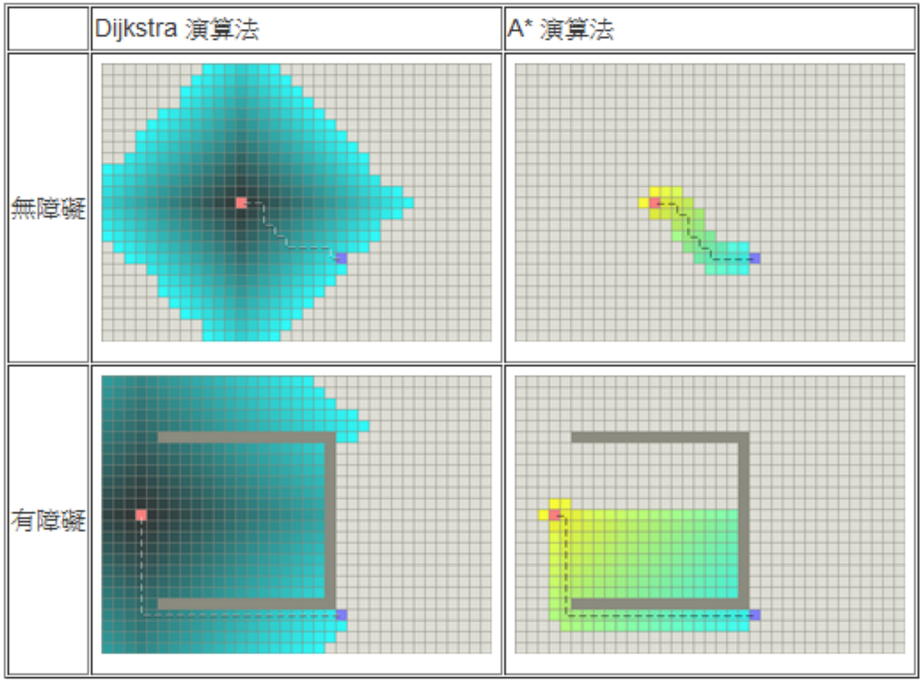
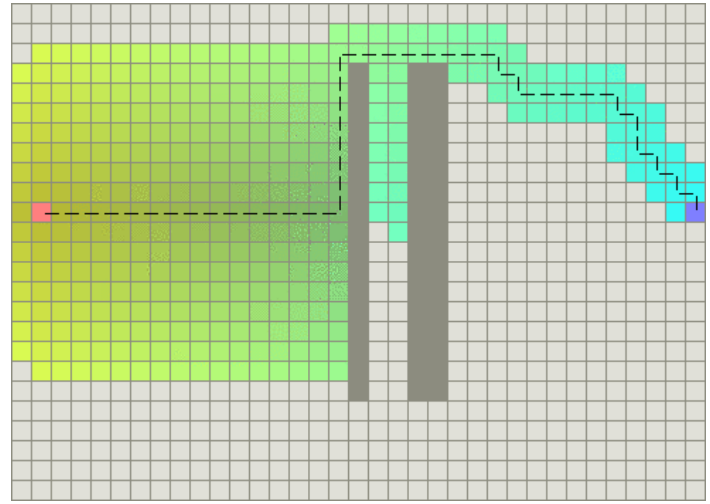
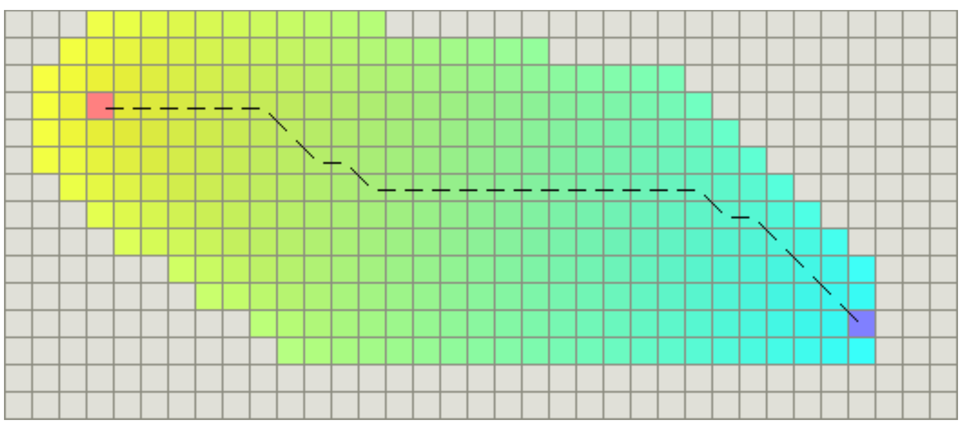


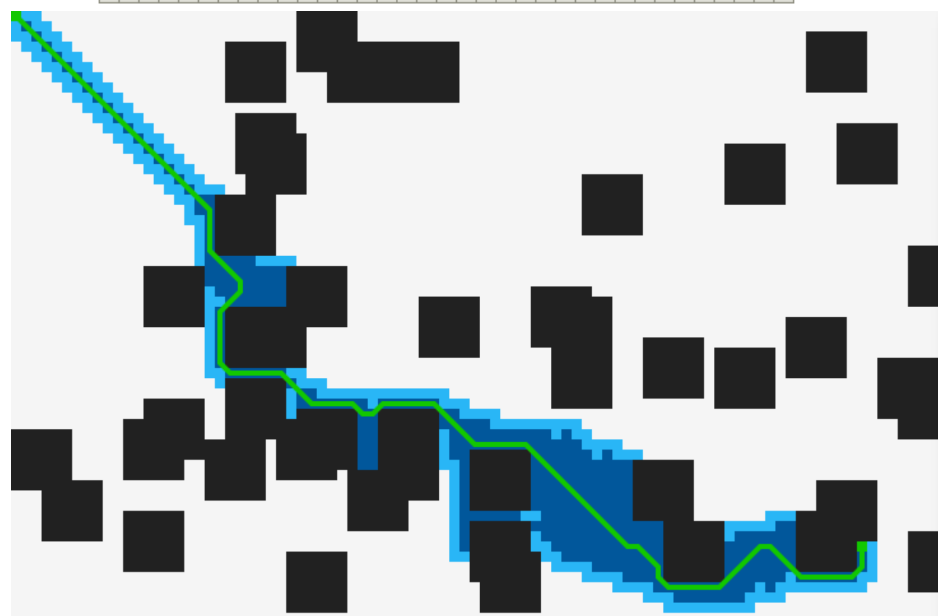
Figure 3.24 Stages in an A* search for Bucharest. Nodes are labeled with $f = g + h$. The h values are the straight-line distances to Bucharest taken from Figure 3.22.

A* demonstrations









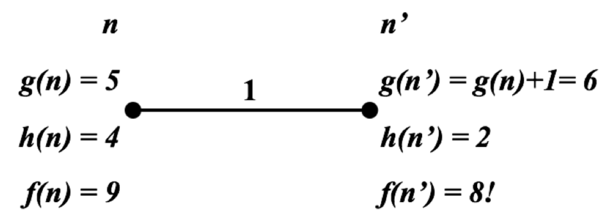
A* Optimality

- *A** search is *complete* and *optimal* under two conditions
  - The heuristic must be *admissible*
  - The costs along a given path must be *monotonic*

- A heuristic h is admissible iff $h(n) \leq h^*(n)$, for all n
  - $h^*(n)$ is the *actual* path-cost from n to the goal
- i.e. h must never over-estimate the cost
  - e.g. h_{SLD} never over-estimates

- A heuristic h is monotonic iff $h(n) \leq c(n, a, n') + h(n')$, for all n, a, n'
  - n' is a successor to n by action a
  - This is basically the triangle inequality
  - n to the goal "directly" should be no more than n to the goal via any successor n'
- Pathmax modification: $f(n') = max(g(n')+h(n'), f(n))$
- Note that optimal here means "finds the best goal"
- We are not arguing that h itself is optimal in any sense

- We want to avoid this sort of situation:

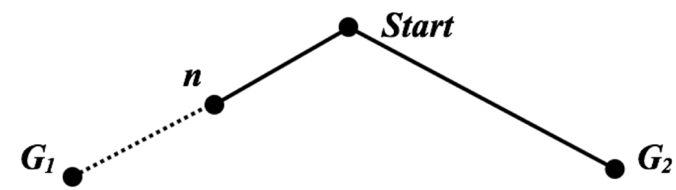


A* proof of optimality

- To show that A^* is optimal, it is sufficient to show that no sub-optimal goal is ever visited

Suppose that the optimal goal is G_1, and that the unvisited set contains both

  - A node n on the shortest path to G_1
  - A sub-optimal goal G_2



- We can prove that n is always visited before G_2

$f(G_2) = g(G_2) + h(G_2)$	definition of f
$= g(G_2)$	since G_2 is a goal, $h(G_2) = 0$
$> g(G_1)$	since only G_1 is optimal
$= g(G_1) + h(G_1)$	since G_1 is a goal, $h(G_1) = 0$
$= f(G_1)$	definition of f
$\geq f(n)$	since h is monotonic

- Thus ***all*** nodes on the shortest path to G_1 will be visited before G_2 is visited
  - Which means that G_1 will be visited before G_2

A* viewed operationally

- *A** visits nodes in order of increasing *f*
- It creates *contours* of nodes, "stretching" to the goal
  - cf. breadth-first or uniform-cost search
- If *f** is the actual cost of the optimal solution
  - *A** visits all nodes *n* with *f(n)* < *f**
  - And it visits some nodes *n* with *f(n)* = *f**

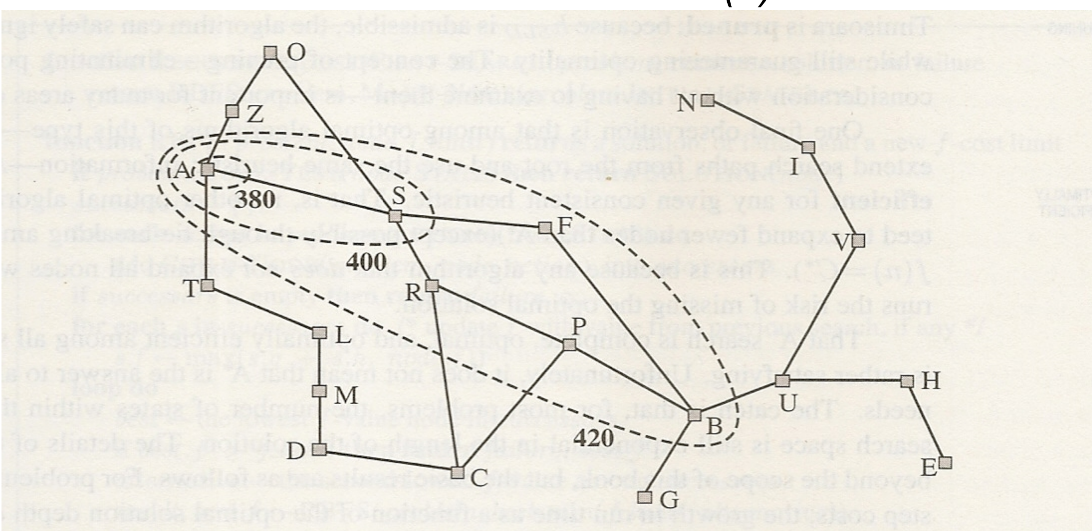


Figure 3.25 Map of Romania showing contours at $f = 380$, $f = 400$, and $f = 420$, with Arad as the start state. Nodes inside a given contour have f-costs less than or equal to the contour value.

Performance of A*

If *x* is the number of nodes *n* with *f(n)* ≤ *f**

- Complete: yes, unless *x* is infinite
- Optimal: yes
- Time: O(*x*)
- Space: O(*x*), keeps all nodes in memory

Clearly *x* depends on the quality of the heuristic…

Assessing Heuristics

- Straight-line distance is an obvious heuristic for travel
  - And it is obviously admissible
- Consider again the 8-puzzle
- A heuristic should be defined so that nodes/states which are "closer to the goal" return smaller values
- Two possible heuristics are
  - $h_1(n)$ = the number of misplaced tiles
  - $h_2(n)$ = the total Manhattan distance of all tiles
- $h_1(s_0) = ?$
- $h_2(s_0) = ?$
- Is either or both admissible?
  - How can we compare them?

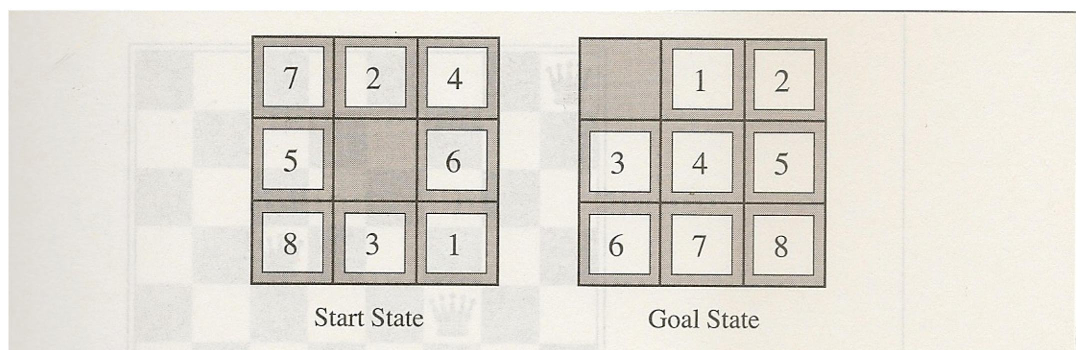


Figure 3.4 A typical instance of the 8-puzzle.

Heuristic Quality

- The quality of a heuristic can be expressed as its *effective branching factor*
- Assume A^* visits N nodes, and finds a solution at depth d
- The effective branching factor b^* is the branching factor of a “perfect tree” with these measurements
- i.e. $N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$
- b^* tends to be fairly constant over problem instances
  - therefore b^* can be determined empirically
- A good heuristic would have b^* close to 1
- h_2 beats h_1, which beats uninformed
  - But is this always true?

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 3.29 Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle for each of various solution lengths d.

Heuristic dominance

- We say that h_2 dominates h_1 iff they are both admissible, and $h_2(n) \geq h_1(n)$, for all nodes n
  - i.e. $h^*(n) \geq h_2(n) \geq h_1(n)$

- If h_2 dominates h_1, then A^* with h_2 will usually visit fewer nodes than A^* with h_1
- The "proof" is obvious
  - A^* visits all nodes n with $f(n) < f^*$
  - i.e. it visits all nodes with $h(n) < f^* - g(n)$
  - f^* and $g(n)$ are fixed
  - So if $h(n)$ is bigger, n is less likely to be below-the-line

- Normally you should always favour a dominant heuristic
  - The only exception would be if it is computationally much more expensive…

- But suppose we have two admissible heuristics, neither of which dominates the other
  - We can just use both!
  - $h(n) = max(h_1(n), h_2(n))$
  - Generalises to any number of heuristics

Deriving heuristics

- Coming up with good heuristics can be difficult
  - So can we get the computer to do it?
- Given a problem *p*, a *relaxed* version *p'* of *p* is derived by reducing restrictions on operators
  - Then the cost of an exact solution to *p'* is often a good heuristic to use for *p*
- e.g. if the rules of the 8-puzzle are relaxed so that a tile can be moved anywhere in one go
  - h_1 gives the cost of the best solution
- e.g. if the rules are relaxed so that a tile can be moved to any adjacent space (whether blank or not)
  - h_2 gives the cost of the best solution
- We must always consider the cost of the heuristic
  - In the extreme case, a perfect heuristic is to perform a complete search on the original problem
- Note that in the examples above, no searching is required
  - The problem has been *separated* into eight independent sub-problems

Deriving heuristics cont.

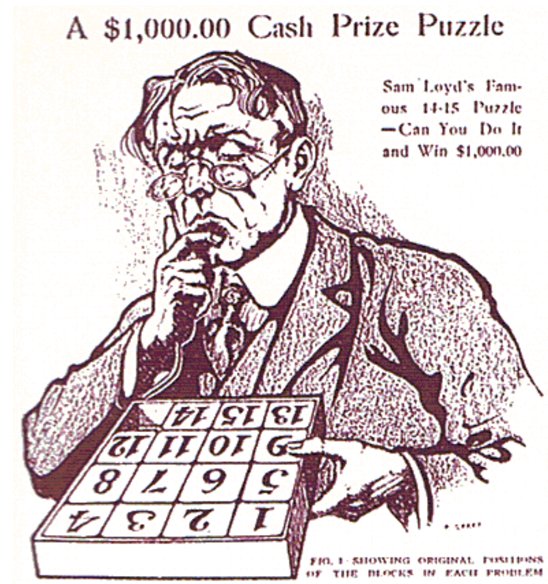
- If a problem is expressed in a suitable formal language, relaxation can be performed automatically
- The 8-puzzle operator is defined by

adjacent(A, B) & blank(B) $\rightarrow$ *canmove(A, B)*

- We can relax the problem by eliminating one or more conditions

adjacent(A, B) $\rightarrow$ *canmove(A, B)* (h_2)

blank(B) $\rightarrow$ *canmove(A, B)* (h_1)



- e.g. Absolver [Prieditis 1993]
  - Discovered a new heuristic for the 8-puzzle, better than any previous one
  - Discovered the first useful heuristic for the Rubik's cube

Memory bounded A*

- The limiting factor on what problems *A** can solve is normally space availability
  - cf. breadth-first search
- We solved the space problem for uninformed strategies by iterative deepening
  - Basically trades space for time, in the form of repeated calculation of some nodes
- We can do the same here
  - *IDA** uses the same idea as ID
  - But instead of imposing a depth cut-off, it imposes an *f*-cost cut-off
- *IDA** performs depth-limited search on all nodes *n* such that $f(n) \leq k$
  - Then if it fails, it increases *k* and tries again
- *IDA** suffers from three problems
  - By how much do we increase *k*?
  - It doesn't use all of the space available
  - The only information communicated between iterations is the *f*-cost limit

Simplified Memory-Bounded A*

- *SMA** implements *A**, but it uses all memory available
- *SMA** expands the most promising node (as in *A**) until memory is full
  - Then it must drop a node in order to generate more nodes and continue the search
- *SMA** drops the least promising node in order to make space for exploring new nodes
  - But we don't want to lose the benefit of *all* the work that has already been done…
  - It is possible that the dropped node may become important again later in the search
- When a node *x* is dropped, the *f*-cost of *x* is backed-up in *x*'s parent node
  - The parent thus has a lower bound on the cost of solutions in the dropped sub-tree
  - Note this again depends on admissibility
- If at some later point in the search, all other nodes have higher estimates than the dropped sub-tree, it is re-generated
  - Again, we are trading space for time

SMA* example

- Each node is labelled with its *g* + *h* = *f* values, and the goal nodes (*D*, *F*, *I*, *J*) are shown in squares. ***The memory can hold only three nodes.***
- Each node is labelled with its current *f*-cost, and values in parentheses store the values of the best dropped descendants.
- At each stage, a successor is added to the deepest, lowest *f*-cost node that has successors not in the tree. Add *B* to *A*.
- *A* is more promising than *B*, so add *G* to *A*. Now we have completely expanded *A*, we update its *f*-cost to 13, the minimum of its descendants. Memory is now full.
- *G* is more promising than *B*, but room must be made first. We drop the shallowest, highest *f*-cost leaf, i.e. *B*. Store *B*'s *f*-cost in its parent *A*, and add *H* to *G*. But *H* is not a goal, and it fills up memory, so we cannot find a goal down that path. Set *H* to ∞.
- Drop *H* and store its *f*-cost in *G*; add *I* to *G*; update *G*'s *f*-cost; update *A*'s *f*-cost. *I* is a goal with cost 24, but because *A*'s cost is 15, there may be a more promising one.
- *A* is once again the most promising, so drop *I* and re-generate *B*.
- Drop *G* and store its *f*-cost in *A*; add *C* to *B*. *C* is not a goal, and it fills up memory, so set *C* to ∞.
- Drop *C* and store its *f*-cost in *B*; add *D* to *B*; update *B*'s *f*-cost; update *A*'s *f*-cost.
- *D* is now the deepest, lowest *f*-cost node, and because it is a goal, the search is complete.

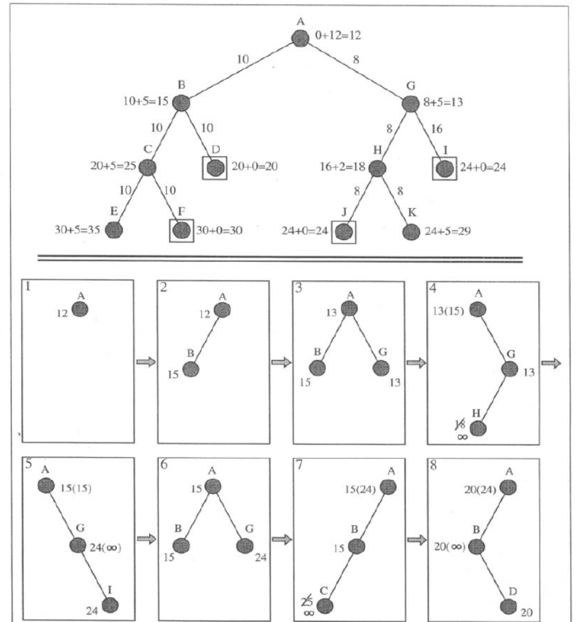


Figure 4.11 Progress of an SMA* search with a memory size of three nodes, on the state space shown at the top. Each node is labelled with its *current* *f*-cost. Values in parentheses show the value of the best forgotten descendant.

SMA* performance

- Complete: yes, if any solution is reachable with the memory available
  - i.e. if a linear path to the depth *d* can be stored
- Optimal: yes, if an optimal solution is reachable with the memory available, o/w returns the best possible
- Time: O(*x*), *x* being the number of nodes *n* with $f(n) \le f^*$
- Space: all of it!

- In very hard cases, *SMA** can end up continually switching between candidate solutions
  - i.e. it spends a lot of time re-generating dropped nodes
  - cf. thrashing in paging-memory systems
- But it is still a robust search process for many problems