1. Find the corresponding Hopfield net with binary values \{0, 1\} to solve the eight rooks problem where each rook has to be positioned in a different row and column to the other in order to avoid a mutual attack. Hint: guess first the pertinent energy to be minimized and then transform it to the canonical Hopfield energy form (a two dimensional multi-flip-flop problem). Does the canonical energy form give the correct answer for the net architecture, weights and biases? If not, change them accordingly.

2. Show that for the 3 neuron Hopfield net with biases \( \theta_1 = \theta_2 = \theta_3 = 0.5 \) and synaptic weights \( \omega_{12} = \omega_{21} = \omega_{32} = \omega_{23} = 1, \omega_{13} = \omega_{31} = -1 \) and \( \omega_{11} = \omega_{22} = \omega_{33} = 0 \) (see also Example 2 from Lecture on Hopfield Nets Part B) we can reach global minimum \((-1, -1, -1)\) of the corresponding energy function e.g. by updating the net asynchronously at \( t = 2 \) the second neuron, at \( t = 3 \) the first neuron, at \( t = 4 \) the second neuron and at \( t = 5 \) the third neuron. We assume that at \( t = 1 \) the initial net state is \((1, -1, 1)\). How can we be sure that \((-1, -1, -1)\) is a stable state?

3. Consider partially specified non-threshold Boolean function \( f \) defined as \( f(1, 1, 1) = 1, f(0, 1, 0) = 0, f(0, 0, 0) = 1, f(1, 0, 0) = 1 \) and \( f(0, 0, 1) = 0 \). Follow the Kashyap Th. from the lecture (see also the pertinent lecture example) and construct the separating \( \phi \)-surface. Verify whether the \( \phi \)-surface determined by the equation \( \phi(x_1, x_2, x_3) = 0 \) correctly separates the function \( f \).

4. (a) Show that partially defined Boolean switching function: \( f(0, 0, 0) = 1, f(1, 0, 0) = 0, f(0, 1, 0) = 0, f(1, 1, 0) = 1, f(0, 1, 1) = 1 \) and \( f(1, 1, 1) = 1 \) is not linearly separable function. Use at least two alternative schemes to prove it (e.g. unateness).

(b) Are XOR and not(XOR) comparable Boolean Switching functions?

(c) Add to \( f \) defined in (a) \( f(0, 0, 1) = 1 \) and \( f(0, 1, 0) = 1 \). Show by using monotonicity criterion that \( f \) is not linearly separable. Consider now \( g(x_1, x_2) = AND(x_1, x_2) \). Show first by using complete monotonicity criterion and then by summability test that \( g \) is linearly separable. Which criterion is stronger and why?

5. Apply Kashyap Theorem to the linearly separable Boolean Switching function such as \( AND(x_1, x_2) \). Show that the resulting polynomial is not a linear separator. Infer now the applicability of the Kashyap Th. for the LTG and PTG gates.

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