Circle Geometry, v1.1

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1. Given a circle with radius \( x \) whose centre is at \( a, b \), and a circle with radius \( y \) that touches \( x \):
   \( y \)'s centre \( c, d \) satisfies \((c - a)^2 + (d - b)^2 = (x + y)^2\), i.e. \( c = a \pm \sqrt{(x + y)^2 - (d - b)^2}\)
   (or conversely \( d = b \pm \sqrt{(x + y)^2 - (c - a)^2}\)).

2. Given a circle with radius \( x \) whose centre is at \( 0, 0 \), a circle with radius \( y \) that touches \( x \) and whose centre is at \( 0, d \), and a circle with radius \( z \) that touches both:
   \( z \)'s centre \( e, f \) satisfies \( f = d^2 + (z + x)^2 - (z + y)^2\), \( e = \pm \sqrt{(z + x)^2 - f^2}\).

Given a circle with radius \( x \), and a tangent to \( x \):

1. if a circle on the tangent with radius \( y \) touches \( x \),
   their centres’ separation along the tangent is \( z = 2\sqrt{xy}\),

1a. conversely, a touching circle on the tangent whose centre has horizontal separation \( z \)
   has radius \( y = \frac{z^2}{4x} \),

2. the biggest circle that fits between \( x \) and \( y \) on the tangent has radius
   \( \frac{xy}{x + y + 2\sqrt{xy}} \).

Generalising (2), given two circles with radii \( y \) and \( w \) whose centres are separated by \( u \) along a common tangent, the biggest circle that fits between them on the tangent has radius
\( \frac{u^2}{4\sqrt{(y + w)^2}} \).

Given a circle with radius \( x \) whose centre is \( z \) from a right-angled corner:

1. the biggest circle that fits in the corner on \( x \)'s side has radius \( z + 2x - 2\sqrt{x(z + x)} \),

2. a touching circle whose centre is \( y \) from the corner around from \( x \) has radius \( w = \frac{y^2 - 2xy + z^2}{2(z + x)} \),

2a. conversely, if a touching circle with radius \( w \) is around the corner from \( x \),
   its centre is \( y = x + \sqrt{x^2 - z^2 + 2w(x + z)} \) from the corner.