CITS5501 Software Testing and Quality Assurance
Formal methods

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Sources

- Pressman, R., *Software Engineering: A Practitioner's Approach*, McGraw-Hill, 2005
- Huth and Ryan, *Logic in Computer Science*
- Pierce et al, *Software Foundations vol 1*
- Alloy tutorial at http://alloytools.org

Specification languages

Some problems in specifying systems

System specifications can suffer from a few potential problems.

- *Contradictions*. In a very large set of specifications, it can be difficult to tell whether there are requirements that contradict each other.
    - Can arise where e.g. specifications are obtained from multiple users/stakeholders
    - Example: one requirement says “all temperatures” in a chemical reactor must be monitored, another (obtained from another member of staff) says only temperatures in a specific range.

Some problems in specifying systems

- *Ambiguities*. i.e., statements which can be interpreted in multiple different ways.

  "The operator identity consists of the operator name and password; the password consists of six digits. It should be displayed on the security screen and deposited in the login file when an operator logs into the system."

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  "The operator identity consists of the operator name and password; the password consists of six digits. It should be displayed on the security screen and deposited in the login file when an operator logs into the system."

- . . . Does "it" refer to the identity, or the password?

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- "is tall" is vague: some people are definitely tall, and some are definitely short, but it can be difficult to tell when exactly someone meets the criterion of being tall.
- Likewise "is user-friendly", "should be secure", "straightforward to understand" are all vague.

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- e.g. An obviously incomplete requirement: “A user may specify normal or emergency mode when requesting a system shutdown. In normal mode, pending operations shall be logged and the system shut down within 2 minutes.”
- . . . So what happens in emergency mode?
- But other cases of incompleteness may be harder to spot.

Some problems in specifying systems

- *Mixed levels of abstraction*. Mixing very high-level, abstract statements with very low-level ones. This makes it difficult to distinguish the high-level architecture from low-level details.

Formal specifications

- Formal specifications can help with ameliorating these problems.
- Sometimes, just the process of attempting to formalize a requirement can reveal problems with it.
- Using a formal model can help reveal *ambiguity* and *vagueness* and allow them to be eliminated
- It may also be possible (depending on the mathematical model used) to detect inconsistencies
- Detecting whether a specification is *complete* is more difficult.
  - Some gaps may be able to be detected
  - But there are nearly always some details that are left undefined, or scenarious that may not have been considered.

Formal specification languages

Many *specification languages* exist, which allow aspects of a system to be modellled mathematically.

Some examples:

- **Z notation**
  - based on set theory and predicate logic
  - developed in the 1970s.
  - Now has an ISO standard, and variations (e.g. object-oriented versions)
- **TLA+**:
  - Stands for "Temporal Logic of Actions"
  - A general-purpose specification language
  - Especially well-suited for writing specifications of concurrent and distributed systems
  - For finite state systems, can check (up to some number of steps) that particular properties hold (e.g. safety, no deadlock)

Formal specification languages

- We'll be using the **Alloy** specification language
- Alloy is both a language for describing structures, and a tool (written in Java) for exploring and checking those structures.
- Influenced by Z notation, and modelling languages such as UML (the Unified Modelling Language).
- Website: http://alloy.mit.edu/

Alloy language

- We'll look at a simple model of a file system (based on the Alloy tutorial at http://alloytools.org/tutorials/)
- To a first approximation, Alloy looks a little like Java:

```
// A file system object in the file system
sig FSObject { parent: lone Dir }

// A directory in the file system
sig Dir extends FSObject { contents: set FSObject }

// A file in the file system
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  you can think of a signature as defining an "entity".
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- Comments can be written in multiple ways
  - single-line comments with "`//`" or "`--`"
  - multile comments with "`/* ... */`"
- **`sig`** is used to write a *signature declaration*; you can think of a signature as defining an "entity". It says what kinds of things there are.
- To be precise, it defines a *set* of things. **`sig FSObject`** defines a set named **`FSObject`**. Which in this case represents the set of all *file system objects* (both files and directories)

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- When we declare **Dir** or a **File** to be sub-types of **FSObject**, they are considered to be *mutually disjoint* sets
  - i.e., nothing is both a **Dir** or a **File**

Alloy – facts

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Alloy – facts

- How can we express that any **FSObject** is either a **Dir** or a **File**? (i.e., there are no other sorts of **FSObject**)
- Alloy also allows us to specify *constraints*. These are introduced with the keyword **fact**.

```
sig FSObject { parent: lone Dir }
sig Dir extends FSObject { contents: set FSObject }
sig File extends FSObject { }

// All file system objects are either files or directories
fact { File + Dir = FSObject }
```

Alloy – facts

- The general syntax for a fact is

  `fact` *name* { *formulas* }

- *formulas* are Boolean expressions, and by putting them in a fact, we're constraining them to be true.

Alloy – abstract signatures

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Alloy – abstract signatures

- (An alternative way to say that all FSObjects must be Dirs or Files would be to declare FSObject **abstract**)
- (This is similar to the use of the **abstract** keyword in Java; it means there are no objects that are *directly* of type FSObject; they must be members of some subtype, instead.)

Alloy – operators

Operators are available to construct Boolean expressions.

- subset: **`in`**
  - *set1* **`in`** *set2* — *set1* is a subset of *set2*
  - informally: "some *set2* are *set1*", or "a *set2* may be *set1*"; but the set-theoretic meaning is more precise.
- set equality: **`=`**
  - *set1* **`=`** *set2* — *set1* equals *set2*
- scalar equality: **`=`**
  - `scalar` **`=`** *value* — *scalar* equals *value*

Alloy – example signatures and facts

- “There are things called animals”

```
sig Animal {}
```

- “A cat is a sort of animal”

```
sig Cat extends Animal {}
```

Exercise

Games:

- There are things called games.
- Games can be board games, or field games.
- There may be other sorts of games.

Alloy – subsets

- We saw that subtypes are disjoint.
- We can also declare sub*sets*:

  sig *signame* **in** *supername* { ... }

- Subsets are *not* necessarilly disjoint, and may have multiple parents

Alloy – subsets

```
sig Animal {}
sig Cat extends Animal {}
sig Dog extends Animal {}
sig FurryPet in Cat + Dog {}
```

- "`FurryPet`" is a subset of the union of `Cat` and `Dog`.
- Some dogs and cats may not be furry (hairless breeds).
- We could *make* them all furry as follows:

```
fact { Cat + Dog = FurryPet }
```

- Are there animals other than cats and dogs?
  Can they be furry?

More operators

- We can use Boolean connectives **`and`**, **`or`**, **`implies`**, **`iff`**, **`not`** to join Boolean expressions.
- e.g.

```
fact { A + B = C and X + Y = Z }
```

Relations

- In our file-system example, we also saw things in the *body* of signatures (i.e., between the braces).
- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
  ```

Relations

- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
- To a first approximation, we can think of relations as behaving like *fields* in an OO language.
- `sig FSObject { parent: lone Dir }` can be read as "Things of type `FSObject` *have a parent*, which is of type `Dir`".
- **`lone`** means "at most one" – i.e., you can have zero or one parents. (We need this because the root directory has no parent.)
  ```

Relations

- ```
  // A file system object in the file system
  sig FSObject { parent: lone Dir }

  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }

  // A file in the file system
  sig File extends FSObject { }
  ```
- More precisely, `parent` is a relation between `FSObject` and `Dir`.
  ```

Relations – multiplicities

- **`lone`** is a type of multiplicity – it says how many of something there are.
- Other multiplicities:
  - **`one`** - one
  - **`some`** - at least one; one or more
  - **`set`** - zero or more
  - **`no`** - zero
- The default multiplicity is one.

Relations – multiplicities

- In set theory terms . . .
- **`one`** means the relation is a total function –
  `sig Student { name : one String }` –
  for every Student, we can map to a string which is their name.
- **`lone`** means the relation is a *partial* function –
  `sig Student { driverLicenseNum : lone String }` – \
  for every Student, we *may* be able to map to a diver's license number.
  (Here, it's assumed you can't have more than one license.)

Relations

- So, signature declarations will look like:

```
sig SomeName {
  field1 : FieldType,
  field2a, field2b : OtherFieldType
}
```

- The order of declarations doesn't matter – `SomeName`, `FieldType` and `OtherFieldType` could be declared in any order in a file.

Relations

- ```
  // A directory in the file system
  sig Dir extends FSObject { contents: set FSObject }
  ```
- Here, we say that a `Dir` has a field `contents`, which is a *set* of `FSObjects`.
- The could contain one item, many items, or no items.
  ```

Examples

- "A car has one engine"

  `sig Car { engine: one Engine }`, or
  `sig Car { engine: Engine }`

- "People have zero or more hobbies"

  `sig Person { hobbies: set Activity }`

Exercises

- Classes have at least one lecturer, and zero or more students.

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Exercises

- Classes have at least one lecturer, and zero or more students.
- Animals have zero or more legs
- Some animals are carnivores
- Textbooks have one or more pages

Back to the file system example

```
sig FSObject { parent: lone Dir }

sig Dir extends FSObject { contents: set FSObject }

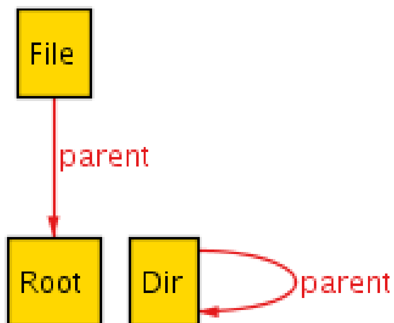
sig File extends FSObject { }

// There exists a root
one sig Root extends Dir { } { no parent }
```

- FSObjects have parents, and directories have contents, and we have constrained the multiplicities . . .
- but there's currently no connection between them.

File system

- So we could have this situation:



File system

- We will need to constrain things more, so we'll use a *fact*.

```
// A directory is the parent of its contents
fact { all d: Dir, o: d.contents | o.parent = d }
```

- This says: "for any thing (let's call it *d* for the moment) of type `Dir`, and for any thing (let's call it *o* for the moment) which is in the set `d.contents`:
  o's parent is *d*.
- It uses a *quantifier* ("all") – we'll look at these more later.