Lecture 21: 3D Modelling: Subdivision Surfaces
Objectives

• Introduce fundamental techniques for creating 3D models,

• in particular subdivision surfaces for easily creating curved surfaces.
Why do we need 3D modelling?

• So far we’ve seen how to draw 3D models while mostly ignoring where they come from,

• but models need to come from somewhere.

• 3D Models can come from three main sources
  1. Scanning real objects
  2. Non-rigid linear deformation of real object scans
  3. Making synthetic models in 3D modelling softwares (CAD)
Scanning Real Objects

• Many 3D scanners are available in the market. Price depends on the resolution of the scan. Examples are
  - Minolta Vivid laser scanner & 3dMD face scanner
  - Microsoft Kinect ($200), PrimeSense, Realsense etc

• To cover complete $360^\circ$, we must scan the object from multiple directions and then stitch them together

• Real scans need a lot of post-processing to remove noise, spikes and cover holes
Non-Rigid Deformation of Linear Models

• Deformable models are made by aligning 3D models of many real objects.

• By changing the parameters of the deformable model, we get different 3D shapes that are linear combinations of the original objects.

• You have already seen how this works in MakeHuman:
  - MakeHuman is used in the project to vary the body shape, skin colour, and facial shape.
  - MakeHuman also provides a small selection of clothing and a skeleton.
Computer Generated Models

• Scanning real objects and making deformable models are outside the scope of this unit

• We will focus only on how to generate 3D models using computer software

• MakeHuman

• And **Blender**
  - Blender includes many different tools useful for different kinds of modelling.
  - We'll focus only on a couple of fundamental techniques: subdivision surfaces and animation via “skinning”.
  - In the project, we have given instructions for how to import real animations (motion capture) into Blender to animate a model
How can we easily model in 3D?

3D modelling can be tedious and time consuming.

– Even positioning a single point in 3D is tricky – Mice and displays are 2D devices

– OpenGL (and DirectX) is based mostly on drawing many triangles.

– So objects must be constructed from many vertices, edges and faces,

– Placing each vertex/edge/face individually is not usually feasible!

– How can we do this quickly and easily?
Can we easily model natural shapes?

We can quickly model “blocky” objects – with only a few faces.

– But most natural shapes aren’t blocky.
  
  We can use prebuilt common shapes like spheres, cylinders, elipsoids, ...

– But these still don’t allow us to create “natural” shapes – most shapes in the real world aren’t perfect spheres, etc.

– Can we generate shapes with many vertices by controlling just a few?
Subdivision surface method is a method for producing smooth surfaces that can be adjusted easily.

- The idea is to specify a blocky surface, with a manageable number of faces and to calculate a smooth surface that roughly follows it.
- The smoothing process needs to be predictable.
- It is related to earlier techniques, like NURBS (Non-Uniform Rational B-Splines) which also use a small number of control points.
  - Subdivision surfaces is better for 3D modelling because it doesn’t have as strict requirements, such as the points forming a grid of quadrilaterals.
  - It is also useful to be able to edit the mesh at the different levels of subdivision, which isn’t possible with NURBS and similar techniques.
Catmull-Clark subdivision surface technique is often the preferred technique for generating smooth surfaces from a “control mesh” with a relatively small number of points, because it is simple, predictable and has desirable properties such as:

- Each original point affects only a small part of the surface – roughly up to each neighbour.

- The 1-st derivative is always continuous – i.e., the normals never change suddenly.

- The 2-nd derivative is nearly always continuous, i.e., the curvature (rate of change of the normals) doesn’t change suddenly. The exception is at extraordinary vertices – where the mesh is "irregular", i.e., not a grid of quadrilaterals (marked in blue in the figure.)
Working with Subdivision Surfaces

Meshes should usually have mostly quadrilaterals
- Having mostly a grid around the mesh makes it easy to adjust.
- It also tends to smooth well.
- One way to create meshes like these is by “extruding a cube” as in the youtube tutorial in part 2 of the project (also see other blender tutorials).

To keep things manageable, we work with relatively few control points.
- Just enough to accurately create the desired smooth shape.
- Loop subdivisions are often an easy way to add a few points when needed.
- To add many new vertices, subdivide the whole mesh one level.
- You can also select edges to subdivide, although this can cause irregularity. (It’s better to do whole areas or loops at once.)
- Apply the smoothing subdivision just before exporting, or before adding an armature for skinning animation.
- To easily swap between different levels of subdivision, use the multiresolution modifier, which remembers fine detail while editing earlier subdivision levels.
- For 3D detail, try the sculpt tool.
Subdivision surfaces: technical details

See the following two papers:


These papers cover more detail than is required for this unit. We’ll focus on the main aspects of the design of the technique, and why it works well.
Subdivision surfaces: technical details

[From Catmull & Clark]

\( o = \) old vertices \( (p_{ij}) \)
\( x = \) new vertices \( (q_{ij}) \)

After one subdivision step, there is a new vertex for:
• Each old face
• Each old edge
• Each old vertex

Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh
Subdivision surfaces: technical details

[From Catmull & Clark]

\( o = \text{old vertices } (p_{ij}) \)

\( x = \text{new vertices } (q_{ij}) \)

After one subdivision step, there is a new vertex for:

- Each old face o
- Each old edge o
- Each old vertex ⊗

Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh
Subdivision surfaces: technical details

[From Catmull & Clark]

There is a new vertex for:
• Each old face

On the old surface, there are 9 faces. So there are 9 new vertices. These 9 new vertices are marked as ○.

(e.g. $p_{11}p_{12}p_{22}p_{21}$ is a face on the old surface)

Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh
Subdivision surfaces: technical details

[From Catmull & Clark]

There is a new vertex for:
• Each old edge

On the old surface, there are 12 edges. So there are 12 new vertices. These 12 new vertices are marked as ○.

Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh
Subdivision surfaces: technical details

[From Catmull & Clark]

There is a new vertex for:
• Each old vertex

On the old surface, there are 4 vertices: \( p_{22}, p_{23}, p_{33}, p_{32} \). So there are 4 new vertices. These 4 new vertices are marked as .

Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh
Subdivision surfaces: technical details

[From Catmull & Clark]

\( o = \text{old vertices} \ (p_{ij}) \)
\( x = \text{new vertices} \ (q_{ij}) \)

After one subdivision, there is a new vertex for:

- Each old face
- Each old edge
- Each old vertex

So, in total, the new surface has \(9 + 12 + 4 = 25\) vertices.

*Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh*
Subdivision surfaces: technical details

Let’s refer to the new vertices as points and the old vertices as vertices.

[From Catmull & Clark]

- New “face” points are at the average of the vertices for the face.
- New “edge” points are at the average of the two vertices on the edge and the two face points on either side of the edge.
- New “vertex” points are more complicated (PTO).

*Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh*
Subdivision surfaces: technical details

Let’s refer to the new vertices as points and the old vertices as vertices

*From Catmull & Clark*

For the vertex $P$, a new point is placed at

$$\frac{F + 2E + (n - 3)P}{n}$$

Where $F$ is the average of the face points, $E$ is the average of the edge points and $n$ is the number of edges.

The faces and edges are the original ones that touch the original $P$.

*Figure 1. Standard bicubic B-spline patch on a rectangular control-point mesh*
Other important properties of *Catmull & Clark* subdivision:

- Where the control points form a simple grid topology (as in Figure 1) the surface tends towards a bicubic B-Spline, a standard kind of surface used when smoothness is required.
- Unlike other techniques for generating such surfaces (like **NURBS**), the technique naturally extends to other topologies, giving 3D modellers much freedom.
- Properties like texture coordinates can be smoothly generated in a similar way to the vertex positions: by averaging them with the same weights during subdivision.

**NURBS = Non-Uniform Rational Basis Spline**
Counting the number of new vertices for open surfaces after one subdivision step can be a bit confusing. For closed surfaces, the counting is easier and more intuitive.

Exercise: Consider a cube.
- After one subdivision step, how many vertices are there?
- After two subdivision steps, how many vertices are there?
Verify your result using the program blender.
Catmull & Clark subdivision surface method on closed surfaces

• Exercise (answer): For a cube, initially

There are 8 vertices, 6 faces, and 12 edges.
\[ V_0 = 8 \]
\[ F_0 = 6 \]
\[ E_0 = 12 \]

- After one subdivision step, how many vertices are there?

There are 26 vertices, 24 faces, and 48 edges.
\[ V_1 = V_0 + F_0 + E_0 = 26 \]
\[ F_1 = 4F_0 = 24 \]
\[ E_1 = 2E_0 + 4F_0 = 24 + 24 = 48 \]

- In general, after \( n \) subdivision steps,

\[ V_n = V_{n-1} + F_{n-1} + E_{n-1} \]
\[ F_n = 4F_{n-1} \]
\[ E_n = 2E_{n-1} + 4F_{n-1} \]
Catmull & Clark subdivision surface method on closed surfaces

- Thus, after two subdivision steps:
  there are $V_2 = V_1 + F_1 + E_1 = 26 + 24 + 48 = 98$ vertices.

- As the Catmull & Clark subdivision surface method constrains the surface to be smooth, the cube would approach the shape of a sphere after a few subdivisions.

- The result above can be verified in the software blender.
Subdivision Examples in Blender

Starting with a cube, after 1 subdivision step (left figure) and after 2 subdivision steps (right figure).
Further Reading

• E. Angel and D. Shreiner: Interactive Computer Graphics 6E © Addison-Wesley 2012
  - Catmull-Clark subdivision Ch-10 Section 10.12

• Using Blender to make a smooth gingerbread man starting with simple cubes
  - https://www.youtube.com/watch?v=bblqJCFsiP0&app=desktop

• Blender is capable of making models with fine details e.g. Bruce Willis
  - https://www.youtube.com/watch?v=zIfRNVe1kmQ