1. For each of the following binary relations on the natural numbers (i.e. \( R \subseteq \mathbb{N} \times \mathbb{N} \)) state whether the relation is one to many, one to one, many to many or many to one. Give a brief reason for each answer.

   (a) \\{ (1, 2), (1, 4), (1, 6), (2, 3), (4, 3) \}

   (b) \\{ (9, 7), (6, 5), (3, 6), (8, 5) \}

   (c) \\{ (12, 5), (8, 4), (6, 3), (7, 12) \}

   (d) \\{ (2, 7), (8, 4), (2, 5), (7, 6), (10, 1) \}

2. Characterise (as 1-1, 1-M, M-1, M-M) each of the following binary relations \( R \) on the given sets. Give a brief reason for each answer.

   (a) \( R \subseteq \mathbb{N} \times \mathbb{N} \) where \( R = \{ (x, y) \mid x = y + 1 \} \)

   (b) \( R \subseteq \mathbb{R} \times \mathbb{R} \) where \( R = \{ (x, y) \mid x = 5 \} \)

   (c) \( R \subseteq \text{females} \times \text{females} \) where \( R = \{ (x, y) \mid \text{daughterof}(x, y) \} \)

   (d) \( R \subseteq \text{people} \times \text{people} \) where \( R = \{ (x, y) \mid \text{daughterof}(x, y) \} \)

For questions 3 to 5 you should understand the four properties of relations and be able to prove which properties a given relation has. The properties are: R reflexive, T transitive, S symmetric, A anti-symmetric. It is usually a good idea to first try to look for evidence (one counter-example is enough) that shows a property does \textit{not} hold.

3. Which properties (R,T,S,A) does each of the following binary relation \( R \) on the set \( A = \{0, 1, 2, 4, 6\} \) have? Give reasons for your answer.

   \( R = \{ (0, 0), (1, 1), (2, 2), (4, 4), (6, 6), (0, 1), (1, 2), (2, 4), (4, 6) \} \)

4. Which properties does each of the following binary relation on the set \( \mathbb{N} \) have?

   \( R = \{ (x, y) \mid \text{even}(x \times y) \} \)

   (That is, \( x \) times \( y \) is an even number)

5. Consider the relation \( \subseteq \) defined on the power set \( \mathcal{P}(\{1, 2, 3\}) \) (recall that the power set of \( A \) is the set of all subsets of \( A \)). Give reasons to support your answers for each of the following.

   (a) Is \( \subseteq \) reflexive?

   (b) Is \( \subseteq \) transitive?

   (c) Is \( \subseteq \) symmetric?
(d) Is $\subseteq$ antisymmetric?

6. Draw the following relations as bipartite graphs (ie. a graph showing edges between only a,b pairs in the relation - see the course notes for details)

(a) Your timetabled classes and day of the week e.g. (cits2211,Mon) and (cits2211,Thu) are in the relation.

(b) the relation divides on the set $\{1, 2, 3, \ldots, 10\}$. We say a divides b if and only if b is a multiple of a. For example, $(2, 6) \in \text{divides}$ but $(2, 5) \notin \text{divides}$.

7. (challenge) Relations can be used to model preferences using $x \geq y$ to mean that $x$ is “at least as good as” $y$ or I prefer $y$ over $x$.

Consider 1000 cups of coffee, numbered C0, C1, C2, \ldots up to C999. Cup C0 contains no sugar, cup C1 one grain of sugar, cup C2 two grains etc. Since one cannot taste the difference between C999 and C998, they are equally good (of equal value), C999 $\geq$ C998 or C999 is no worse than C998. For the same reason, we have C998 $\geq$ C997, etc. all the way up to C1 $\geq$ C0.

Since preference is transitive, we should have C999 $\geq$ C0 which means that C999 is no worse than C0. But clearly C0 is worse than C999. This contradicts transitivity of indifference, and therefore also transitivity of weak preference.

Suggest some possible ways of resolving this paradox?