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1. Given the three premises

- $S \rightarrow L$
- $\neg S \rightarrow \neg J$
- $Y \rightarrow \neg L$

Use propositional logic to prove formally that

- $J \rightarrow \neg Y$

**Solution:**

Let our premises be

- (1) $S \rightarrow L$
- (2) $\neg S \rightarrow \neg J$
- (3) $Y \rightarrow \neg L$

Then the steps in our formal proof will be:

- (4) From (2), $J \rightarrow S$ (by the contrapositive equivalence)
- (5) From (3), $L \rightarrow \neg Y$ (by the contrapositive equivalence)
- (6) From (1) and (4), $J \rightarrow L$ (implication transitivity)
- (7) From (5) and (6), $J \rightarrow \neg Y$ (implication transitivity) ■
2. In the Venn diagrams below, shade in the regions (i) \((A \cap B) \cup C\) and (ii) \(A \cap (B - C)\).

(i) \((A \cap B) \cup C\)

(ii) \(A \cap (B - C)\)
3. For sets $A = \{a, b, c\}$ and $B = \{x, y\}$, write down (using correct bracketing) the following sets:

(i) $A \cap B$
(ii) $A \times B$
(iii) $\mathcal{P}(A)$

**Solution:**

(i) $A \cap B$ is $\emptyset$ or $\{\}$
(ii) $A \times B$ is $\{(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\}$
(iii) $\mathcal{P}(A)$ is $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$
4. Let \( A = \{1, \{1\}, \{2\}, 3\} \).

Decide whether each of the following statements is True (T) or False (F).

(i) \(1 \in A\)
(ii) \(\{3\} \in A\)
(iii) \(\{\{1\}\} \subseteq A\)
(iv) \(3 \subseteq A\)
(v) \(\{1, 2\} \subset A\)
(vi) \((1, 2) \in (A \times A)\)

**Solution:**

(i) \(1 \in A\) – This is true
(ii) \(\{3\} \in A\) – This is false
(iii) \(\{\{1\}\} \subseteq A\) – This is true
(iv) \(3 \subseteq A\) – This is false
(v) \(\{1, 2\} \subset A\) – This is false
(vi) \((1, 2) \in (A \times A)\) – This is false

/ 2
5. (a) Draw the bipartite graph diagram and the directed graph diagram of the relation \( R \) on the set \( X = \{0, 1, 2, 3\} \) defined by \((a, b) \in R\) if and only if \(1 \leq a + b \leq 3\).

(b) Is \( R \) one-to-one, one-to-many, many-to-one, or many-to-many? Give a brief explanation.

(c) For each of the properties reflexivity, symmetry, transitivity and antisymmetry, explain carefully whether \( R \) has that property or not.

**Solution:**

\[
\begin{array}{c}
\text{(a) } \begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
0 & & & \\
1 & & & \\
2 & & & \\
3 & & & \\
\end{array} \\

\text{(b) } \begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
0 & & & \\
1 & & & \\
2 & & & \\
3 & & & \\
\end{array}
\end{array}
\]

\[\begin{array}{c}
\text{(a) } \begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
0 & & & \\
1 & & & \\
2 & & & \\
3 & & & \\
\end{array} \\

\text{(b) } \begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
0 & & & \\
1 & & & \\
2 & & & \\
3 & & & \\
\end{array}
\end{array}\]

(b) many-to-many because (for instance) 0 in the domain is related to many (1, 2 and 3) in the codomain, and vice versa.

**Solution:**

Marking Scheme

Two marks for each correct diagram, one mark for part (b). No marks if the answer is given yes/no with no explanation.
6. Suppose a binary relation $R$ is defined on the set of all integers by the following definition: $(a, b) \in R$ iff $a$ is a multiple of $b$. So for instance, $(6, 2) \in R$ and $(6, -3) \in R$ but $(4, 5) \notin R$.

For each of the properties reflexivity, symmetry, transitivity and antisymmetry, explain carefully whether $R$ has that property or not.

**Solution:**

$a$ is a multiple of $b$ if and only if $\exists m.a = m \times b$.

The relation IS reflexive: for any $a$, $(a, a) \in R$ since $a = a \times 1$, so every $a$ is a multiple of itself.

The relation is NOT symmetric – a counterexample is that $(6, 2) \in R$ but $(2, 6) \notin R$.

The relation IS transitive because if $(a, b) \in R$ and $(b, c) \in R$ then there are some $m$ and $n$ such that $a = m \times b$ and $b = n \times c$ so $a = m \times n \times c$ and thus $(a, c) \in R$.

The relation is NOT antisymmetric. $(2, -2) \in R$, and $(-2, 2) \in R$, and $2 \neq -2$ – but antisymmetry requires that if $(a, b)$ and $(b, a)$ are both in $R$, then $a = b$.

(Alternatively you might say: if $a \neq b$, then only $(a, b)$ or $(b, a)$ can be in $R$, but not both. However, both $(2, -2)$ and $(-2, 2)$ are in $R$, so it isn’t antisymmetric.)

Marking Scheme:

Two marks for each of the correct response with a clear and correct explanation.

General Feedback:

The most common mistake was thinking that the relation IS anti-symmetric. But it is not. This relation would be anti-symmetric on the natural numbers, but it is not anti-symmetric on the integers, as shown above.