1. Use a truth table to prove or disprove the following statement.

\[ \neg((P \lor Q) \land R) \text{ is logically equivalent to } (\neg P) \land (\neg Q \land \neg R) \]
2. Which of the following are tautologies? Give brief reasons for each answer.

(a) \( P \rightarrow P \)
(b) \( (P \lor Q) \rightarrow P \)
(c) \( (P \lor \neg P) \rightarrow P \)
(d) \( (Q \land ((P \land Q) \lor (\neg P \land \neg Q))) \rightarrow P \)
(e) \( (P \land \neg P) \rightarrow P \)
3. Consider the following four propositions forming an argument.

   If Tiexia is sick then he is absent.
   If Tiexia is absent then there will be no test with his name on it.
   There is a test with Tiexia’s name on it.
   Therefore, Tiexia is not sick.

   Using variables for simple propositions and some technique that we have studied show that the conclusion must be true if the first three propositions are.
4. Recall that conjunction distributes over disjunction, i.e.

\[(A \lor B) \land C \text{ is logically equivalent to } (A \land C) \lor (B \land C)\]

Consider the following formula of propositional logic.

\[(P_{1,1} \lor P_{1,2} \lor P_{1,3}) \land (P_{2,1} \lor P_{2,2}) \land (P_{3,1} \lor P_{3,2})\]

Here each \(P_{i,j}\) is just a simple proposition which is true or false.

This formula has 19 symbols as you count the \(P_{i,j}\) as just one symbol (and counting parentheses).

This is called a conjunction (\(\land\)) of disjunctions (\(\lor\)).

Find a logically equivalent formula which is a disjunction of conjunctions. You don’t need to write it all out but tell us how many symbols it has.
5. Express the following colloquial English statements using predicate logic. (First identify the predicates.)

(a) All data scientists have application domain knowledge.
(b) No hackers have application domain knowledge.
(c) Some software developers are data scientists.
(d) No data scientists are hackers.
(e) Although all data scientists are software developers, some are hackers if they do not have application domain knowledge.
6. This question is about the set of all integers. Suppose that we colour in some integers red and some green. Write $R(n)$ or $G(n)$ in those cases.

Translate the following properties into normal English.

(a) $G(0)$

(b) $\forall x. \neg (G(x) \land R(x))$

(c) $\exists x. \exists y. \exists z. (x < y \land y < z \land G(x) \land R(y) \land G(z))$

(d) $\forall x. (G(x) \rightarrow (\neg \exists y. (x < y \land R(y)) \lor (\forall y. (x < y \rightarrow R(y))))$